ON IYENGAR'S TAUBERIAN THEOREM FOR NÖRLUND SUMMABILITY

O. P. VARSHNEY

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1. Let $\sum a_n$ be a given infinite series with the sequence of partial sums $\{s_n\}$. Let $\{p_n\}$ be a sequence of real, non-negative constants, and let us write

$$P_n = p_0 + p_1 + \cdots + p_n.$$

The sequence-to-sequence transformation:

$$t_n = \frac{p_0 s_n + p_1 s_{n-1} + \cdots + p_n s_0}{P_n} \quad (P_n \neq 0) \tag{1. 1}$$

defines the sequence $\{t_n\}$ of Nörlund means of the sequence $\{s_n\}$ generated by the sequence of coefficients $\{p_n\}$. The sequence $\{s_n\}$ is said to be summable (N, p_n) to the sum s, if $\lim_{n\to\infty} t_n$ exists and equals s [1, p. 64].

In the special case in which p_n is defined by

$$\sum_{n=0}^{\infty} p_n x^n = \left(\sum_{0}^{\infty} \frac{x^n}{n+1}\right)^k = \left\{\frac{\log(1-x)^{-1}}{x}\right\}^k, \ |x| < 1, \ (k = 1, 2, \cdots), \tag{1. 2}$$

t_n reduces to the familiar 'Harmonic mean of order k.' In this case we say that the sequence $\{s_n\}$ is summable (H, k) to the sum s. The case $k = 1$ is due to Riesz [6] and the general case is due to Iyengar [2]. It should be noted that Iyengar writes (N, k) in place of (H, k). We do not use his notation to avoid confusion with (N, p_n) where $p_n = k$.

The main interest of this method of summation lies in the following Tauberian theorem:

THEOREM 1 [2]. *If s_n $(n = 0, 1, \cdots)$ is a sequence summable (H, k) to 0, and if*

$$s_n - s_{n-1} \equiv a_n \leqq A n^{-\mu} \quad (A > 0, \ 1 > \mu > 0), \tag{1. 3}$$

then $s_n \to 0$.

Rajagopal [4] has given a simple proof of this theorem in the case $k = 1$ which has been adapted by Rangachari [5] to the general case.

Iyengar [3], after proving the case $k = 1$ of Theorem 1, states without proof the following extension which is interesting in view of the application which he has made of it, to obtain a scale of convergence tests for Fourier