A CHARACTERIZATION OF CONTACT TRANSFORMATIONS

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1. Introduction. Let M^{2n+1} be a contact manifold with a contact form η [2], [3]. By definition, η satisfies the relation

$$(1) \qquad \eta \wedge \overbrace{d\eta \wedge \cdots \wedge d\eta}^{n} \neq 0,$$

where d means the exterior differentiation and $\wedge$ means the exterior multiplication. The Pfaffian equation

$$(2) \qquad \eta = 0$$

determines in M^{2n+1} a $2n$-dimensional distribution D which we shall call the *contact distribution*. We say that a tangent vector X of M^{2n+1} belongs to the distribution D if and only if

$$(3) \qquad \eta(X) = 0$$

is satisfied.

An r-dimensional submanifold F in M^{2n+1} is said to be an *integral submanifold* (of the contact distribution D) if and only if every tangent vector of F at every point p of F belongs to D. An integral submanifold of dimension r in M^{2n+1} is said to be a *maximal intergral submanifold* if it is not a pure subset of any other integral submanifold of dimension r.

A diffeomorphism of M^{2n+1} is said to be a *contact transformation* [3] of M^{2n+1} if and only if

$$(4) \qquad f_{*}\eta = \sigma\eta$$

holds good, where f_{*} is the dual map of f which acts on the vector space of 1-forms of M^{2n+1} and σ is a function over M^{2n+1} which does not vanish at any point of M^{2n+1}.

The purpose of this paper is to prove Theorem B which characterizes contact transformations. Theorem A on the highest dimension of integral submanifold of D is given as a preliminary of Theorem B.