LEBESGUE CONSTANTS FOR A FAMILY OF SUMMABILITY METHODS

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1. Introduction. The unboundedness of the sequence of Lebesgue constants implies the existence of a continuous function whose Fourier series diverges at a point, and this is also the case with many summability methods. The estimation of such constants for various summability methods has been calculated by K. Ishiguro [2], [3], A. E. Livingston [4], and L. Lorch [5], [6], [7], [8].

In this paper we shall study the behavior of the Lebesgue constants for a family of summability methods. A. Meir [9] has introduced a family of summability methods which is defined by two parameters a and q, and has shown that this family contains Borel, Valiron, Euler, Taylor and S_α-transformation.

If we define $L_F(a, q(p))$ by the Lebesgue constants for this family of summability methods, then we obtain the following formula:

$$(1.1) \qquad L_F(a, q(p)) = \frac{2}{\pi^2} \log 4aq(p) + A + O(\log q/\sqrt{q}\,) \quad \text{as } p \to \infty\,,$$

where A is the constant such as

$$(1.2) \qquad A = -\frac{c}{\pi^2} + \frac{2}{\pi}\int_0^1 \frac{\sin u}{u}\, du - \frac{2}{\pi}\int_1^\infty \left(\frac{2}{\pi} - |\sin u|\right)\frac{du}{u}$$

and

$$c = \int_0^1 \frac{1-e^{-u}}{u} - \int_1^\infty \frac{1}{ue^u}\, du\,,$$

which is Euler-Mascheroni's constant.

The proof of formula (1.1) consists of two parts; 1°) the case where $q = q(p)$ is integer and 2°) the case where $q = q(p)$ is not integer. In the last section we shall show that from (1.1) we can obtain Lebesgue constants for Borel, Valiron, Euler, Taylor and S_α-transformation which are contained in this family of summability methods.

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