ON GENERALIZED CESÀRO MEANS OF INTEGRAL ORDER

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1. Introduction. It is the object of this paper to consider some of the properties of a λ-type generalization (C, λ, κ) of Cesàro summability, which reduces to (C, κ) when $\lambda_n = n$. We shall be concerned mainly with the relations between (C, λ, κ) and other summability methods, notably the Riesz method (R, λ, κ) and a more general method (G, λ) defined by means of a function g. Except in this introductory section, we shall deal almost entirely with methods of integral order (we draw attention to this by writing p in place of κ), and we suppose throughout that $\lambda = \{\lambda_n\}$ is a sequence satisfying

$$0 \leqslant \lambda_0 < \lambda_1 < \cdots < \lambda_n \to \infty .$$

Given any series[2] $\sum a_\nu$, and any $\kappa \geqslant 0$, denote

$$A^\kappa(\omega) = \sum_{\lambda_\nu < \omega} (\omega - \lambda_\nu)^\kappa a_\nu ;$$

if

$$\omega^{-\kappa} A^\kappa(\omega) \to s \quad \text{as} \quad \omega \to +\infty$$

then we say that $\sum a_\nu$ is Riesz summable (R, λ, κ) to s. When $\omega \to \infty$ through the sequence $\{\lambda_n\}$, we obtain the definition of 'discontinuous' Riesz summability (R^*, λ, κ), and we may then relax the restriction on κ to $\kappa > -1$; thus $\sum a_\nu$ is summable (R^*, λ, κ) to s if $\lambda_n^{-\kappa} A^\kappa(\lambda_n) \to s$.

It is of course trivial that[3], for any $\{\lambda_n\}$ and any $\kappa \geqslant 0$,

$$(R, \lambda, \kappa) \subseteq (R^*, \lambda, \kappa) .$$

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2) Unless otherwise specified, limits of summation or integration are assumed throughout to be 0, ∞.

3) Given two summability methods A, B, we say that A is included in B (written $A \subseteq B$) if every series summable-A is also summable-B (to the same value); A and B are equivalent (written $A \sim B$) if each includes the other.