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ON THE ABSOLUTE SUMMABILITY FACTORS OF POWER SERIES

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1. Let

$$f(z)=\sum_{n=0}^{\infty} c_n z^n=\sum_{n=0}^{\infty} c_n r^n e^{in\theta}$$

be a function regular for $r=|z|<1$. If for some $p>0$, the integral

$$\int_0^{2\pi} |f(re^{i\theta})|^p\, d\theta$$

remains bounded when $r\to 1-0$, the function $f(z)$ is said to belong to the class H^p. It is well known that, if $f(z)$ belongs to the class H^p, then $f(z)$ has a boundary value $f(e^{i\theta})=\lim_{r\to 1} f(re^{i\theta})$ for almost all θ $(0\leqq\theta\leqq 2\pi)$ and $f(e^{i\theta})$ is integrable L^p. Moreover if $p\geqq 1$, a necessary and sufficient condition for the function $f(z)$ to belong to the class H^p is that

$$\sum_{n=0}^{\infty} c_n e^{in\theta}$$

is the Fourier series of its boundary function $f(e^{i\theta})$.

Let us denote

$$s_n(\theta)=s_n(\theta;f)=\sum_{u=0}^{n} c_\nu e^{i\nu\theta},$$

$$t_n(\theta)=t_n(\theta;f)=nc_n e^{in\theta},$$

$$\sigma_n^\alpha(\theta)=\sigma_n^\alpha(\theta;f)=(C,\alpha)\text{ mean of the sequence }\{s_n(\theta)\}$$

$$=\frac{1}{A_n^\alpha}\sum_{\nu=0}^{n} A_{n-\nu}^{\alpha-1}s_\nu(\theta)\qquad \textit{for } \alpha>-1,$$

and