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AN EXTENSION OF AN APPROXIMATION PROBLEM PROPOSED BY K. ITÔ

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The following problem, which first was proposed by Itô, is of some interest to the theory of probability:

If $f(n)$ is a sequence with

$$\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} |f(n)|^2 < \infty , \tag{1}$$

does there always exist a polynomial $P(x)$ such that

$$\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} |f(n) - P(n)|^2 < \varepsilon$$

for any assigned $\varepsilon > 0$?

Izumi [2] has given an affirmative answer if (1) is strengthened to

$$\sum_{n=0}^{\infty} |f(n)|^2/w^n < \infty \quad \text{for some} \quad w > 0 .$$

A more general existential proof, based upon the Hahn-Banach Theorem is due to Edwards [1].

In this paper we shall obtain a very short and simple proof of the following extension of Itô's problem:

THEOREM. *Suppose that*

$$\int_0^{\infty} H(|f(t)|)\, d\alpha(t) < \infty , \tag{2.1}$$

$f(t)$ continuous for $t \geqslant 0$, $H(t) \geqslant 0$ continuous and not decreasing for $t \geqslant 0$,