Tôhoku Math. Journ.
21(1969) 47-55.

SUBNORMAL OPERATOR WITH A CYCLIC VECTOR

TAKASHI YOSHINO

(Received June 28, 1968)

In this paper, we aim to characterize the non-trivial closed invariant subspace of a subnormal operator and to study the existence of such subspaces.

An operator A acting on a Hilbert space H is said to be subnormal if, on some space K containing H, there exists a normal operator B such that $Bx=Ax$ for every x in H; then B is called a normal extension of A.

The normal extension B, acting on K, of a subnormal operator A, acting on H, a subspace of K, is the minimal normal extension of A if the smallest subspace of K that contains H and reduces B is K itself; Halmos has shown that any two minimal normal extensions are unitarily equivalent ([3]).

If A is subnormal on H, we call that a vector x is cyclic with respect to A if the smallest subspace containing x and invariant under A is H; in this case we say that H is cyclic with respect to A.

For our purpose, it is natural to assume that the subnormal operator A on H has a cyclic vector x; because, if $\vee\{A^n x;\ n \geqq 0\}$ (which denotes the smallest closed subspace containing $A^n x;\ n \geqq 0$) is properly included in H, then it is clearly a non-trivial closed invariant subspace of A.

Bram proved in [1] that when A is normal and acts on H, the fact that H is cyclic with respect to A in the sense just defined is equivalent to the fact that H is cyclic in the usual sense, i.e., that there exists x in H such that H is the smallest closed subspace that contains x and reduces A.

It is known that if B is a normal operator on K with a cyclic vector, then there exists a unitary mapping U of K onto a suitable function space $L^2(d\mu(\lambda); \sigma(B))$ such that UBU^{-1} has the form of "multiplication by λ" ([2]).

Applying this representation theorem of normal operators to the minimal normal extension B on K of a subnormal operator A on H with a cyclic vector, we can show that H admits a representation relative to A onto a subspace $H^2(d\mu(\lambda);\ \sigma(B))$ of $L^2(d\mu(\lambda);\ \sigma(B))$.

In the next section, we show this representation of a subnormal operator with a cyclic vector and using this, we give the sufficient conditions of the existence of non-trivial closed invariant subspaces of subnormal operators.

We state here a characterization of subnormal operators given by Halmos [3] and Bram [1] without the proof.