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INVARIANT SUBMANIFOLDS IN A SASAKIAN MANIFOLD

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1. Introduction. The theory of invariant submanifolds in a contact Riemannian manifold was initiated by M. Okumura [5].

In this note we wish to classify invariant submanifolds of codimension 2 which are η-Einstein manifolds in a Sasakian manifold with constant ϕ-sectional curvature.

To state the main theorem we prepare the followings. An odd dimensional Euclidean space E^{2n+1} (resp. an odd dimensional sphere S^{2n+1}) has the standard Sasakian structure with constant ϕ-sectional curvature $H=-3$ (resp. $H>-3$) [9]. By CD^n, L and (L, CD^n) we denote the open unit ball in a complex n-dimensional Euclidean space C^n, a real line and the product bundle $L\times CD^n$. The (L, CD^n) also has a Sasakian structre with constant ϕ-sectional curvature $H<-3$ [9]. Let Q^{n-1} be an $(n-1)$-dimensional complex quadric in a complex projective space $P^n(C)$ of complex dimension n. By (S, Q^{n-1}) we denote a circle bundle over Q^{n-1}. Then (cf. [1, p. 61]) since Q^{n-1} is a Kaehlerian manifold of restricted type, (S, Q^{n-1}) defines a Sasakian structure [6]. Since Q^{n-1} $(n\geqq 3)$ is Einsteinian, (S, Q^{n-1}) is η-Einsteinian [8]. Henceforth let $\widetilde{M}$ be one of the E^{2n+1}, S^{2n+1} and (L, CD^n) and $\widetilde{B}$ be C^n (if $\widetilde{M}=E^{2n+1}$), $P^n(C)$ (if $\widetilde{M}=S^{2n+1}$) and CD^n (if $\widetilde{M}=(L, CD^n)$). $\widetilde{M}$ is a principal G^1-bundle over $\widetilde{B}$ and G^1 is a circle or a line. $\pi:\widetilde{M}\longrightarrow\widetilde{B}$ denotes the projection. We may prove the following theorem.

THEOREM. i) *S^{2n-1} and (S, Q^{n-1}) are the only connected complete invariant submanifolds in S^{2n+1} which are η-Einsteinian.*

ii) *(L, CD^{n-1}) (resp. E^{2n-1}) is the only connected complete invariant submanifold in (L, CD^n) (resp. E^{2n+1}) which is η-Einsteinian.*

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2. Local results. (ϕ, ξ, η, g) denote the tensors of the Sasakian structure of $\widetilde{M}$. Let M be a connected contact manifold of dimension $2n-1$ which is