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SOME FUNCTION-THEORETIC NULL SETS

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1. Let E be a totally disconnected compact set in the complex z-plane and let G be the complementary domain of E with respect to the extended z-plane. Consider a domain in G whose relative boundary consists of at most a countable number of analytic curves clustering nowhere in G. Such a domain is called a subregion in G. If for any subregion in G there exists no non-constant single-valued bounded analytic function whose real part vanishes continuously on its relative boundary, then the set E is said to be in the class N_B^0.

It is known that if E is of logarithmic capacity zero, then E belongs to the class N_B^0 and that there exists a compact set of positive logarithmic capacity and belonging to N_B^0 (Kuroda [5]).

It is also known that there exists no non-constant single-valued bounded analytic function in the complementary domain of $E \in N_B^0$, that is, N_B^0 is a subclass of the class N_B in the sense of Ahlfors-Beurling [1].

If E is of logarithmic capacity zero, then there exists an Evans-Selberg's potential which is harmonic in G except at $z=\infty$ and whose boundary value at every point of E is positively infinite. Such a function plays an important role to study the covering property of meromorphic functions in G.

In this paper, we shall treat Noshiro's theorem on cluster sets [10] in detail. In §2, by the argument due to Matsumoto [7], we shall give a sufficient condition in order that there exists an analogous function to an Evans-Selberg's potential in the subregion inside G. As its application, in §3 we shall prove a theorem which is an improvement of Noshiro's theorem [10] on cluster sets under the so-called Hervé's condition. §4 is devorted to show that in the theorem, Hervé's condition can not be dropped. In Appendix, Kuroda's criterion for E to be in the class N_B^0 is proved in a correct form.

2. First we shall prove the following.

THEOREM 1. *If E is a compact set of the class N_B^0, then any closed subset E_0 of E is also in the class N_B^0.*