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NOTE ON THE RE-TOPOLOGIZATION OF A SPACE BY A SET OF OPERATORS

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Let $\{T_i\}$ $(i \in A)$ be a set of linear operators in a locally convex vector space E. In [2], M. Nagumo tried to adopt a new topology on E, which makes each T_i be continuous and makes it possible to extend T_i on all elements of E. His method is a generalization of a so-called Lax's negative norms. As the special cases of it we get Schwartz's distribution space and others. The purpose of this paper is to weaken the condition on E supposed by M. Nagumo. It may be said that our proof corresponds naturally to the technique used in the extension of differential operators.

As J. E. Roberts noted in [4], in particular taking a countable set of closed operators in a Hilbert space, we meet with a countably Hilbert space in the sense of Gel'fand [1]. We note here that the converse is also true, more precisely, that the topology of any countably Hilbert space can be constructed as the projective limit by a set of self-adjoint operators in a Hilbert space.

Let E be a separated locally convex vector space and $\{T_i\}$ $(i \in A)$ be a set of linear operators from E to E. We assume that $D(T_i)$, the domain of each T_i, is dense in E and that the set $\{T_i\}$ contains identity operator I. We shall from now onwards take the set of operators satisfying the following condition:

(C). $F = \bigcap_{i \in A} D(T_i')$ is total over E, i.e., the element x of E, for which $\langle f, x \rangle = 0$ for all f of F, is zero element. Here by T_i' we denote the transpose of T_i.

If the set $D(T_i')$ is total over E, T_i is a pre-closed operator. Therefore when the above condition is satisfied, all T_i are pre-closed. For the reflexive space E condition (C) means that F is dense in E', the dual space of E. Here the dual space of E is considered under the strong topology $\beta(E', E)$. We shall denote it by E'_β later.

Our main problem is to introduce a suitable topology in E and to make any T_i continuous from $D_i = D(T_i)$ to E. In that case, space E under such a topology we denote by $\widehat{E}$, and write completion of $\widehat{E}$ by $\widetilde{E}$. Then, under the