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SOME TAUBERIAN THEOREMS CONCERNING (S^*, μ) TRANSFORMATIONS

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1. Introduction. The regular series to sequence (S^*, μ) transform of a series $\sum_{i=0}^{\infty} a_i$ is defined as follows :

$$(1.1) \qquad S_n^*(\beta)=\sum_{i=0}^{\infty} a_i \sum_{k=i}^{\infty} \binom{k+n}{n} \int_0^1 (1-t)^k t^{n+1} d\beta(t), n\geqq 0,$$

where $\beta(t)$ satisfies

(1.2) $\beta(t)$ is of bounded variation in $[0, 1]$, $\beta(1)-\beta(0+)=1$ and $\beta(1)=\beta(1-)$.

The series to sequence (S^*, μ) transformation is the series to sequence analogues of the sequence to sequence (S^*, μ) transformation defined by Ramanujan [7] §4. We shall be interested in finding Tauberian estimates of the following form. For a series $\sum_{i=0}^{\infty} a_i$ denote $s_n=\sum_{i=0}^{n} a_i$, then what is the best possible constant A satisfying

$$\limsup_{\lambda\to\infty} |S_{n(\lambda)}^*(\beta)-s_{m(\lambda)}| \leqq A \limsup_{n\to\infty} |na_n|$$

where $n(\lambda)$, $m(\lambda)$ are given functions assuming integral values only, and all series $\sum_{i=0}^{\infty} a_i$ satisfying the Tauberian condition

$$(1.3) \qquad \limsup_{n\to\infty} |na_n| < \infty.$$

What is the best constant B satisfying

$$\limsup_{\lambda\to\infty} |S_{n(\lambda)}^*(\beta)-S_{m(\lambda)}^*(\gamma)| \leqq B \limsup_{n\to\infty} |na_n|$$

where $\gamma(t)$ is another function satisfying (1.2), $n(\lambda)$, $m(\lambda)$ are as before and