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ON THE ABSOLUTE SUMMABILITY OF FOURIER SERIES

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1. Introduction.

Let

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) = \sum_{n=0}^{\infty} A_n(x) \tag{1}$$

and let $s_n(x)$ and $\sigma_n^\alpha(x)$ $(\alpha > -1)$ denote the n-th partial sum and n-th (C, α) mean of Fourier series (1), respectively. If the series

$$\sum_{n=0}^{\infty} |\sigma_n^\alpha(x) - \sigma_{n-1}^\alpha(x)|$$

is convergent, we say that the series (1) is absolutely summable (C, α) or summable $|C, \alpha|$ at the point x.

We have

$$\{\sigma_n^\alpha(x) - \sigma_{n-1}^\alpha(x)\} = \frac{\tau_n^\alpha(x)}{n}$$

where

$$\tau_n^\alpha(x) = \frac{1}{A_n^\alpha} \sum_{k=1}^{n} A_{n-k}^{\alpha-1} k A_k(x)$$

and

$$A_n^\alpha = \binom{n+\alpha}{n}.$$

For $f(x) \in L^p (1 \leqq p < \infty)$ we define

$$\omega_p^{(1)}(t, f) = \sup_{0<h<t} \left\{ \int_{-\pi}^{\pi} |f(x+h) - f(x-h)|^p dx \right\}^{1/p}$$