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SPACES OF MAPPINGS AS SEQUENCE SPACES

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1. Let E be a Banach space whose dual is denoted by E'. A sequence $\{b_i\}$ of elements of E is called a Schauder basis of E if each $x \in E$ can be written in a unique way as $x = \sum t_i b_i$. Our summations are always from 1 to ∞ unless other limits of summation are expressly indicated. If $\{f_i\}$ is the sequence of elements of E' biorthogonal to $\{b_i\}$, i.e., $\langle b_j, f_i \rangle = \delta_{ij}$ with $\delta_{ij}=0$ if $i \neq j$ and $\delta_{ii}=1$, then $x = \Sigma \langle x, f_i \rangle b_i$. The basis $\{b_i\}$ is called shrinking if $\{f_i\}$ is a basis of E'.

Let F be a Fréchet space whose topology is determined by a countable system of seminorms $\{p_k\}_{k=1}^{\infty}$. We denote by $\mathcal{L}(E, F)$ the space of all continuous linear mappings from E into F. Equipped with the topology determined by the seminorms

$$q_k(T) = \sup\{p_k(Tx) : x \in E, \|x\| \leqq 1\}; \; T \in \mathcal{L}(E, F); \; k=1, 2, \cdots,$$

$\mathcal{L}(E, F)$ is a Fréchet space. When E has a Schauder basis $\{b_i\}$, an element $T \in \mathcal{L}(E, F)$ is completely determined by the F-valued sequence $\{Tb_i\}$. We denote by $\mathcal{L}$ the space of F-valued sequences $\{\{Tb_i\} : T \in \mathcal{L}(E, F)\}$. Let $C(E, F)$ be the closed subspace of $\mathcal{L}(E, F)$ consisting of compact mappings and let $C = \{Tb_i\} : T \in C(E, F)\}$.

Dixmier [1] (see also [9], [10]) proves that if E and F are Hilbert spaces then $\mathcal{L}(E, F)$ is the bidual of $C(E, F)$. The dual of $C(E, F)$ is the space of all nuclear mappings from E into F equipped with the trace norm. He points out that this relationship is similar to that between c_0, l^1 and l^∞.

The relationship between these sequence spaces can also be described in a very simple way by means of Köthe spaces: l^1 is the space of all sequence $\{\beta_i\}$ such that for each $\{\alpha_i\} \in c_0$, $\sum |\alpha_i| |\beta_i| < \infty$, i.e., in the notation of Köthe $l^1 = (c_0)^\times$; l^∞ is the space of all sequences $\{\gamma_i\}$ such that for each $\{\beta_i\} \in l^1$, $\sum |\beta_i| |\gamma_i| < \infty$, i.e., $l^\infty = (l^1)^\times = (c_0)^{\times\times}$ (see [4], Chapter 6, §30). On the other hand a compact mapping defined on a Hilbert space vanishes outside a separable subspace ([7], page 202) and a separable Hilbert space has a shrinking Schauder basis.