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ON THE ABSOLUTE SUMMABILITY FACTORS OF INFINITE SERIES

S.M.MAZHAR

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1. Let Σa_n be a given infinite series with s_n as its n-th partial sum. We denote by $\{\sigma_n^\alpha\}$ and $\{t_n^\alpha\}$ the n-th (C,α), $(\alpha>-1)$ means of the sequences $\{s_n\}$ and $\{na_n\}$ respectively. A series Σa_n is said to be summable $|C,\alpha|$ if $\Sigma|\sigma_n^\alpha-\sigma_{n-1}^\alpha|<\infty$ and summable $|C,\alpha|_k$, $k\geqq 1$, $\alpha>-1$ if

$$\sum n^{k-1}|\sigma_n^\alpha-\sigma_{n-1}^\alpha|^k<\infty. \tag{1.1}$$

In view of the well known identity $t_n^\alpha=n(\sigma_n^\alpha-\sigma_{n-1}^\alpha)$, the condition (1.1) can also be written as

$$\sum_1^\infty \frac{|t_n^\alpha|^k}{n}<\infty. \tag{1.2}$$

Let $\{p_n\}$ be a sequence of positive real constants such that $P_n=p_0+p_1+\cdots+p_n\to\infty$ as $n\to\infty$. A series Σa_n is said to be summable $|\overline{N},p_n|$ if $t_n^*\in BV$,

where
$$t_n^*=\frac{1}{P_n}\sum_{k=0}^n p_k s_k.$$

For $p_n=\dfrac{1}{n+1}$ the summability $|\overline{N},p_n|$ is equivalent to the well known summability $|R,\log n,1|$.

For any real α and integers $n\geqq 0$, we define $\overset{\alpha}{\triangle}U_n=\sum_{v=n}^\infty A_{v-n}^{-\alpha-1}U_v$, whenever the series is convergent.

2. It is known that summability $|\overline{N},p_n|$ and the summability $|C,\alpha|_k$ are, in general, independent of each other. It is, therefore, natural to find out suitable summability factors $\{\varepsilon_n\}$ so that $\Sigma a_n\varepsilon_n$ may be summable $|C,\alpha|_k$, $\alpha>-1$, $k\geqq 1$, whenever Σa_n is summable $|\overline{N},p_n|$, and conversely, if Σa_n is summable $|C,\alpha|_k$ then $\Sigma a_n\varepsilon_n$ may be summable $|\overline{N},p_n|$. In a recent paper [5] the author has