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MINIMAL SUBMANIFOLDS AND CONVEX FUNCTIONS

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In this note we describe some results concerning minimal submanifolds in complete Riemannian manifolds of non-negative curvature. Our main theorem is stated as follows:

MAIN THEOREM. *Any compact minimal hypersurface in a complete non-compact Riemannian manifold of non-negative curvature is totally geodesic.*

The author was motivated to study minimal submanifolds by Nakagawa-Shiohama [7] in which they suggested to investigate the relation between compact minimal submanifolds and "souls" of complete non-compact Riemannian manifolds of non-negative curvature. For souls, see Cheeger-Gromoll [2] as well as Shiohama [9]. In the course of our investigation we also deal with a property of the distance function α_N from points on a minimal submanifold N to a totally geodesic hypersurface H in a complete Riemannian manifold of non-negative curvature. Roughly speaking, this property is that α_N is superharmonic on N, which may be seen as the dual of the case where the ambient manifold has non-positive curvature, compare Hermann [6]. Making use of this property we are able to determine the relation between N and H under some conditions, for related results see Frankel [3], [4].

In section 1, we describe some lemmas concerning convex sets and convex functions under fairly general situations. Lemmas 1 and 2 are originally mentioned in Cheeger-Gromoll [2], which we will use without proof. Lemma 4 is an important link in our arguments. In section 2, assuming that the ambient manifold has non-negative curvature, we construct several convex functions to apply Lemma 4. Finally in section 3, we consider the non-compact case to obtain our main theorem, see Nakagawa-Shiohama [7] and Shiohama [8]. Lemmas 5 and 10 are originally proved in Cheeger-Gromoll [2]. For all basic concepts and tools in Riemannian geometry that will be used without comment, we refer to Gromoll-Klingenberg-Meyer [5].

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