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K-THEORY OF LENS-LIKE SPACES AND S^1-ACTIONS ON $S^{2m+1} \times S^{2n+1}$

Dedicated to Professor Shigeo Sasaki on his 60th birthday

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0. Introduction. A smooth S^1-action $\psi: S^1 \times X \to X$ on a smooth manifold X is called *principal* if the isotropy subgroup

$$I(x) = \{z \in S^1 \mid \psi(z, x) = x\}$$

consists of the identity element alone for each point $x \in X$. A principal smooth S^1-action (ψ, X) on a closed (oriented) smooth manifold X is called to *bord* (with orientation) if there is a principal smooth S^1-action (Φ, W) on a compact (oriented) smooth manifold W and there is an equivariant (orientation preserving) diffeomorphism of (ψ, X) onto $(\Phi, \partial W)$, the boundary of W. It is well known that any principal smooth S^1-action on a homotopy sphere does not bord (cf. [3], Theorem 23.2). On the contrary, any principal smooth S^1-action on a closed oriented smooth manifold which is cohomologically a product $S^{2m+1} \times S^{2n+1}$ of odd-dimensional spheres bords with orientation ([7], Theorem 7.3; [5], Theorem 1). Moreover let $(\psi_1, \Sigma_1^{2n+1})$, $(\psi_2, \Sigma_2^{2n+1})$ be any principal smooth S^1-actions on homotopy spheres, then there is an equivariant continuous mapping $f: \Sigma_1 \to \Sigma_2$ and f induces a homotopy equivalence of the orbit manifold Σ_1/ψ_1 to the orbit manifold Σ_2/ψ_2 (cf. [2], Proposition 3.1). On the contrary, there are infinitely many cohomologically distinct principal smooth S^1-actions on $S^{2m+1} \times S^{2n+1}$ $(m \neq n)$ ([5], Corollary of Lemma 2.2).

In this paper we consider the equivariant K-theory of certain S^1-manifolds $S^{2m+1} \times S^{2n+1}$ and we show that there are topologically distinct principal smooth S^1-actions on $S^{2m+1} \times S^{2n+1}$ which can not be distinguished by the cohomology ring structure of the orbit spaces. To state our results precisely, we introduce some notations. Let

$$a = (a_0, a_1, \cdots, a_m)\ , \qquad b = (b_0, b_1, \cdots, b_n)$$

be sequences of positive integers and denote by

$$S^{2m+1}(a_0, a_1, \cdots, a_m) \times S^{2n+1}(b_0, b_1, \cdots, b_n)$$

the product of spheres $S^{2m+1} \times S^{2n+1}$ with the smooth S^1-action $\psi_{a,b}$ defined