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THE CONTACT STRUCTURES ON $\{SU(n+1)\times R/SU(n)\times R\}_\alpha$ AND $\{Sp(n)\times SU(2)/Sp(n-1)\times SU(2)\}_\alpha$ OF BERGER

Dedicated to Professor Shigeo Sasaki on his 60th birthday

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M. Berger has classified all simply connected normal homogeneous Riemannian manifolds of positive curvature ([1]). In odd dimensional case there are seven classes of such structures:

(i) sphere of constant curvature,
(ii) $\{SU(n+1)\times R/SU(n)\times R\}_\alpha$ $(n \geqq 2,\ 0<\alpha\leqq \pi/2)$,
(iii) $\{Sp(n)\times SU(2)/Sp(n-1)\times SU(2)\}_\alpha$ $(n \geqq 2,\ 0<\alpha\leqq \pi/2)$,
(iv) $\{Sp(n)\times R/Sp(n-1)\times R\}_\alpha$ $(n \geqq 2,\ 0<\alpha\leqq \pi/2)$,
(v) $SO(9)/SO(7)$,
(vi) $Sp(2)/SU(2)$,
(vii) $SU(5)/Sp(2)\times S^1$.

It is well known that (i) has a natural contact structure, that is, a Sasakian structure of constant curvature.

In the present note we shall show that (ii) and (iii) have also natural contact structures which relate closely to homogeneous Riemannian metrics, and that these contact structures define S^1 or S^3-fiberings of these manifolds over complex or quaternionic projective spaces. For example in case of (ii) we have the following.

THEOREM. *$\{SU(n+1)\times R/SU(n)\times R\}_\alpha$ has a structure of regualr compact simply connected Sasakian manifold of constant ϕ-holomorphic sectional curvature $4-3/4\cdot\beta^2$, where we have put $\beta=\sqrt{2(n+1)/n}\sin\alpha$. Boothby-Wang's fibering is a principal circle bundle over complex projective space of constant holomorphic sectional curvature 4 (cf. Theorem 2.2).*

1. Preliminaries.

1.1 Let M^{2n+1} be a $(2n+1)$-dimensional Riemannian manifold. Let ∇ denote the covariant differentiation, and $R(X, Y)Z=\nabla_{[X,Y]_M}Z-\nabla_X\nabla_Y Z+\nabla_Y\nabla_X Z$ be the curvature tensor. In the following "[,]" denotes the bracket operation of Lie algebra of Lie group and "$[\ ,\ ]_M$" denotes the bracket of vector fields on the differentiable manifold M.

Now a 1-form η on M^{2n+1} is said to be a contact form if $\eta\wedge(d\eta)^n\neq 0$