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ON THE COMMUTING EXTENSIONS OF NEARLY NORMAL OPERATORS

Dedicated to Professor Masanori Fukamiya on his 60th birthday

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1. We say that a bounded linear operator A on a Hilbert space H is nearly normal if A commutes with A^*A. Recall that if B is a normal operator on K and H is an invariant subspace for B, then the operator $A = B|H$ is said to be subnormal. If the smallest reducing subspace for B containing H is K, then B is said to be the minimal normal extension of A. This is unique to an isomorphism (cf. [3]). It is well-known that every nearly normal operator is subnormal. The purpose of this paper is to give certain necessary and sufficient conditions under which two nearly normal operators on H have the mutually commuting normal extensions on K.

2. For our purpose, we shall consider the following problem: Given a nearly normal operator A on H, and B, its minimal normal extension on K, when can an operator T on H be extended to an operator T^e on K in such a manner that T^e commutes with B? This problem for general subnormal operators was first solved by Bram [1]. We state here it without proof.

PROPOSITION. *Let A on H be a subnormal operator with the minimal normal extension B on K. Then the necessary and sufficient condition that an operator T on H has an extension T^e on K such that T^e commutes with B is that* (a) *T commutes with A, and* (b) *there exists a positive constant c such that for every finite set $x_0, x_1, \cdots, x_r$ in H we have*

$$\sum_{m,n=0}^{r} \langle A^m T x_n, A^n T x_m \rangle \leqq c \cdot \sum_{m,n=0}^{r} \langle A^m x_n, A^n x_m \rangle .$$

If the extension T^e exists, it is unique.

Now we state the following lemmas given in [4] without proof.

LEMMA 1. *If A is a nearly normal operator on H and if E is the projection from H on $\mathscr{N}_A = \{x \in H; Ax = 0\}$, then $E \in R(A) \cap R(A)'$ where*