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NEGATIVE QUASIHARMONIC FUNCTIONS*

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1. The radial quasiharmonic function

$$s(r) = -\sum_{i=0}^{\infty} b_i r^{2i+2} ,$$

defined by $\Delta s = 1$, plays a crucial role in the problem of the existence of bounded quasiharmonic functions on the Poincaré ball $B_\alpha = \{r < 1, ds = (1 - r^2)^\alpha |dx|\}$ (see [18]). In the present paper we shall show that s has the striking property

$$s < 0 \quad on \quad B_\alpha \quad for\ every \quad \alpha .$$

This will lead us to the introduction of the class QN of negative quasiharmonic functions.

We shall carry out our reasoning for dimension $M = 3$. This is the essential case, as for $M = 2$ the harmonicity and the Dirichlet integral are independent of α. We conjecture that the reasoning developed in this paper will allow a generalization to an arbitrary M.

2. We start by stating our main result:

THEOREM 1. *The radial quasiharmonic function* $s(r) = -\sum b_i r^{2i+2}$ *belongs to* QN.

The proof will be given in Nos. 3-12.

3. First we determine the coefficients b_i.

LEMMA 1. *The function*

$$s(r) = -\sum_{i=0}^{\infty} b_i r^{2i+2} \tag{1}$$

with $\Delta s = 1$ *on* B_α *has*

$$b_0 = \frac{1}{6} , \tag{2}$$

and the other coefficients are determined by the recursion formula

$$b_i = p_i b_{i-1} + q_i . \tag{3}$$

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