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A NOTE ON CONFORMAL MARTINGALES

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1. In a forthcoming paper, P. A. Meyer establishes by a very nice method that $(H^1)^* = BMO$ for right continuous martingales, but he does not deal with conformal martingales. The purpose of this note is to extend the fundamental results given by R. K. Getoor and M. J. Sharpe [1] on conformal martingales to locally square integrable martingales under the assumption such that (F_t) has no times of discontinuity. Our proof is an adaptation of the proof due to Getoor and Sharpe.

2. The reader is assumed to be familiar with the basic notions of the theory of stochastic integrals relative to martingales as given in [2].

By a system (Ω, F, F_t, P) is meant a complete probability space (Ω, F, P) with an increasing right continuous family $(F_t)_{t\geqq 0}$ of sub σ-fields of F. We assume as usual that F_0 contains all P-null sets. Let $\mathscr{M} = \mathscr{M}(F_t)$ (resp. $\mathscr{M}_c(F_t)$) be the class of all right continuous (resp. continuous) L^2-bounded martingales X over (F_t) such that $X_0 = 0$. Denote by $\mathscr{M}^{loc}(F_t)$ the class of all locally square integrable martingales X over (F_t) such that $X_0 = 0$.

For each $X \in \mathscr{M}^{loc}(F_t)$, we define:

$$\begin{aligned} \|X\|_H &= E[\langle X, X\rangle_\infty^{1/2}] \\ \|X\|_B^2 &= \sup_t \operatorname*{ess.sup}_\omega E[\langle X, X\rangle_\infty - \langle X, X\rangle_t | F_t] \\ H^1 &= \{X \in \mathscr{M}^{loc} \mid \|X\|_H < +\infty\} \\ BMO &= \{X \in \mathscr{M}^{loc} \mid \|X\|_B < +\infty\} . \end{aligned}$$

Clearly $BMO \subset \mathscr{M}(F_t) \subset H^1$. BMO is a normed linear space with the norm $\|\cdot\|_B$. H^1 is also a normed linear space with the norm $\|\cdot\|_H$, but it should be noted that this is not the same H^1-space introduced by P. A. Meyer for right continuous martingales. Probably, our H^1-space is not complete.

The next inequality is proved in [1] only for continuous H^1-martingales.

THEOREM 1. *For every* $X \in \mathscr{M}^{loc}$

$$\|X\|_H \leqq \sup\{E[\langle X, Y\rangle_\infty]; 0 \leqq \langle X, Y\rangle_\infty \ \ and \ \ \|Y\|_B \leqq 1\} .$$