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ON THE DIVERGENCE OF REARRANGED WALSH SERIES II

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Concerning the rearrangement of Walsh series, the author [2] proved the following

THEOREM A. *If $\{\rho(n)\}$ is a sequence of positive numbers with $\rho(n) = o(\sqrt[4]{\log n})$, then there exists a sequence of real numbers $\{a_n\}$ for which*

$$\sum_{n=1}^{\infty} a_n^2 \rho^2(n) < \infty$$

and such that the Walsh series

$$\sum_{n=1}^{\infty} a_n w_n(x) \tag{1}$$

can be rearranged into an almost everywhere divergent series.

This is a generalization of a theorem in [3], and also can be considered as a Walsh series analogue of a theorem in [1] which concerns the rearrangement of trigonometric series. In the present note, we intend to sharpen further this theorem. We write $L_1(n) = \log n$ and $L_s(n) = L_1(L_{s-1}(n))$ $(s = 2, 3, \cdots)$.

THEOREM. *For any natural number s, there exists a sequence of real numbers $\{a_n\}$ for which*

$$\sum_{n=N+1}^{\infty} a_n^2 \sqrt{L_1(n)}\, L_2(n) L_3(n) \cdots L_s(n) < \infty^{1)}$$

such that the Walsh series (1) *can be rearranged into an almost everywhere divergent series.*

COROLLARY. *For any natural number s, there exists a sequence of real numbers $\{a_n\} \in l_2$ such that the Walsh series* (1) *can be rearranged to satisfy*

$$\limsup_{N\to\infty} \left| \sum_{j=1}^{N} a_{n(j)} w_{n(j)}(x) \right| \{L_1(N)\}^{-1/4} \{L_2(N) L_3(N) \cdots L_s(N)\}^{-1/2} > 0$$

almost everywhere.

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1) N is a natural number depending s such that $L_s(N) > 0$.