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EXISTENCE OF ALMOST PERIODIC SOLUTIONS BY LIAPUNOV FUNCTIONS

F. NAKAJIMA

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1. Introduction. The existence of almost periodic solutions of almost periodic systems has been studied by many authors. Generally, the existence of a bounded solution does not imply the existence of almost periodic solutions [4]. To obtain almost periodic solutions, we need additional conditions, for example, separation conditions and stability conditions. Another approach is to assume the existence of a Liapunov function with some properties ([2], [5]). Relationships between separation conditions and stability conditions have been discussed by the author [3].

In this paper, by assuming the existence of some Liapunov function, we shall obtain an existence theorem for an almost periodic solution, which improves Fink and Seifert's result [2] and proves Yoshizawa's result [5] as a corollary.

We denote by R^n the Euclidean n-space and set $R = R^1$ and $R^+ = [0, \infty)$. Let $|x|$ be the Euclidean norm of $x \in R^n$.

2. Theorem and some remarks. Consider the almost periodic system

$$x' = f(t, x) \qquad (\, ' = d/dt) \,, \tag{2.1}$$

where $x, f \in R^n$ and $f(t, x)$ is defined on $R \times D$, D open set of R^n, and is almost periodic in t uniformly for $x \in D$. The following theorem is an improvement of Fink and Seifert's result [2].

THEOREM. *Suppose that the system* (2.1) *has a solution* $\phi(t)$ *such that* $\phi(t) \in K$ *on* R^+, *where* K *is a compact subset of* D, *and assume that there exists a continuous scalar function* $V(t, x)$ *defined on* $R^+ \times D$, *which satisfies the following conditions*:

(i) $V(t, \phi(t))$ *is bounded on* R^+,

(ii) $|V(t, x) - V(t, y)| \leqq L|x - y|$ *for* $x, y \in S$, $t \in R^+$, *where* S *is any compact subset of* D *and* L *may depend on* S,

(iii) $\dot{V}(t, x) \geqq a(|x - \phi(t)|)$, *where* $a(r)$ *is continuous and positive definite and*

$$\dot{V}(t, x) = \varlimsup_{h \to +0} \frac{1}{h} \{V(t + h, x + hf(t, x)) - V(t, x)\} \,.$$