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A THEOREM ON LIMITS OF KLEINIAN GROUPS

HIRO-O YAMAMOTO

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1. Let G be a group of conformal automorphisms of the extended complex plane $\hat{C} = C \cup \{\infty\}$. Every element of G is a Möbius transformation of the form

$$T: z \mapsto \frac{az+b}{cz+d},$$

where a, b, c and d are complex numbers with $ad - bc = 1$. This transformation T is often identified with $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ in $PSL(2, C)$ and, in this case, $a + d$ is called the trace of T and is denoted by trace T.

If there does not exist a sequence of G which converges to the identity under the topology of $PSL(2, C)$, then G is called discrete.

A point $w \in \hat{C}$ is called a limit point of G provided that there exist a point $z \in \hat{C}$ and a sequence $\{T_i\}_{i=1}^{\infty}$ of elements of G such that $T_j \neq T_k (j \neq k)$ and such that $T_i(z) \to w$ as $i \to \infty$. If a point $w \in \hat{C}$ is not a limit point of G, it is called an ordinary point of G. Denote by $\Lambda(G)$ the set of all limit points of G and by $\Omega(G)$ the set of all ordinary points of G. If $\Omega(G)$ is not empty, then G is called a discontinuous group. If the limit set of a discontinuous group G contains more than two points, then G is called kleinian. A discontinuous group not being kleinian is said to be elementary. It is known that a kleinian group contains infinitely many loxodromic elements and the set of attracting fixed points of loxodromic elements in G is dense in $\Lambda(G)$.

An isomorphism ϕ of a kleinian group G_1 onto a kleinian group G_2 is said to be type preserving if $\phi(T)$ is parabolic if and only if T is parabolic.

Let T be a Möbius transformation of the form

$$T: z \mapsto \frac{az+b}{cz+d}, \qquad c \neq 0 .$$

Then we call two circles $I(T)$: $|z + d/c| = 1/|c|$ and $I(T^{-1})$: $|z - a/c| = 1/|c|$ the isometric circles of T and of T^{-1}, respectively. It is known that T maps the exterior of $I(T)$ onto the interior of $I(T^{-1})$. Since the radii of $I(T)$ and $I(T^{-1})$ are both equal to $1/|c|$ and since the distance of the center of $I(T)$ from that of $I(T^{-1})$ equals $|(a + d)/c|$, a necessary and sufficient