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DISCONTINUOUS GROUPS OF AFFINE TRANSFORMATIONS OF C^3

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1. **Introduction.** Let G be a group of affine transformations acting freely and properly discontinuously on C^n. Suppose that C^n/G is compact. Let G_0 be the subgroup of G consisting of translations, which is a normal subgroup of G. Moreover we assume that $H = G/G_0$ is a finite group. Enriques and Severi show that in the case of surfaces i.e., $n = 2$, H is a cyclic group of order d, $d = 1, 2, 3, 4, 6$, [1]. In this paper in the case of $n = 3$ we shall prove the following

THEOREM. *If H is cyclic, then $H \cong Z/d$, $d = 1, 2, 3, 4, 5, 6, 8, 10, 12$. If H is not cyclic but abelian, then $H \cong Z/d_1 \oplus Z/d_2$, $(d_1, d_2) = (2, 2), (2, 4), (2, 6), (2, 12), (3, 3), (3, 6), (4, 4), (6, 6)$. Finally, if H is not abelian, then H is D_4: a dihedral group of order 8.*

2. Let g be an affine transformation of C^n i.e., $gx = A(g)x + a(g)$ where $x \in C^n$, $A(g) \in GL(n, C)$, $a(g) \in C^n$. If g has no fixed points, then at least one eigenvalue of $A(g)$ has to be 1. It is easy to see that if g has no fixed points, then g^m has no fixed points. We call $A(g)$ the holonomy part of g and A a holonomy representation.

PROPOSITION 1. *Let G be the group in Introduction. If K is an abelian subgroup of G with finite index, then G_0 contains K i.e., G_0 is the largest abelian subgroup of G with finite index.*

PROOF. As K is commutative, all the elements of K can be diagonalized simultaneously. Suppose $K - G_0 \neq \emptyset$ and choose $g \in K - G_0$. Then $gx_j = \alpha_j x_j + a_j$, where $\alpha_1 = 1$, $\alpha_n \neq 1$. May assume $a_n = 0$, because otherwise we consider hgh^{-1} instead of g, h being a translation defined by ${}^t(0, \cdots, 0, a_n/(\alpha_n - 1))$. Owing to the commutativity of K this implies that any $g' \in K$ acts like $g'x_n = \beta_n x_n$. Hence C^n/K is not compact, which contradicts the assumption $|G : K| < \infty$.

COROLLARY 1. *Let G' be the group similar to G. If $G \simeq G'$ by an isomorphism φ, then $\varphi G_0 = G'_0$. Hence $H = G/G_0 \simeq H' = G'/G'_0$.*

PROOF. $\varphi(G_0) \subset G'_0$, and $\varphi^{-1}(G'_0) \subset G_0$, by Proposition 1.