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ON CYCLOTOMIC Z_2-EXTENSIONS OF IMAGINARY QUADRATIC FIELDS

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Let $K = \boldsymbol{Q}(\sqrt{-m})$ for a positive square-free integer m. For each $n \geqq 0$, let $\boldsymbol{B}_n$ be the maximal real subfield of the cyclotomic field of the 2^{n+2}-th roots of unity. Let $\boldsymbol{B}_\infty = \bigcup_{n=0}^{\infty} \boldsymbol{B}_n$ and let $K_\infty = \boldsymbol{B}_\infty \cdot K$. Then the extension K_∞/K is called a cyclotomic $\boldsymbol{Z}_2$-extension. Let h_n be the class number of $K_n = \boldsymbol{B}_n \cdot K$ and let 2^{e_n} be the exact power of 2 dividing h_n. Iwasawa proved, in [2] and [3], that there exist an integer $n_0 \geqq 0$ and an integer c such that

$$e_n = \lambda n + c \quad \text{for all} \quad n \geqq n_0 , \tag{1}$$

where λ is the invariant of this $\boldsymbol{Z}_2$-extension.

The group-theoretic meaning of this invariant λ is as follows. Let A_n be the 2-Sylow subgroup of the ideal class group of K_n. For each $m \geqq n \geqq 0$, the norm map from K_m to K_n defines a morphism from A_m to A_n. Let X be the limit of this projective system, then as an abelian group

$$X \cong \boldsymbol{Z}_2^{\lambda} \oplus T , \tag{2}$$

where T is a finite abelian 2-group. This integer λ coincides with that of (1).

We always define the natural action of $\Gamma = \mathrm{Gal}(K_\infty/K)$ on X and call X the Iwasawa module for K_∞/K as a Γ-module. The action of Γ will be used in Section 4.

In this paper, we shall determine the right hand side of (2), especially the invariant λ, and find a value of n_0 satisfying (1).

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After this paper was accepted for publication, the author received the preprint by B. Ferrero entitled "The cyclotomic $\boldsymbol{Z}_2$-extension of imaginary quadratic fields" in which he proves the same formula for the invariant λ by a purely algebraic method. Moreover, his Theorem 5 c) and f) implies the torsion subgroup T in our Theorem 1 is in fact of order 2.