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ON THE MAJORANT PROPERTIES IN $L^p(G)$

To the memory of the late Professor Karel deLeeuw

E. T. Y. LEE* AND GEN-ICHIRÔ SUNOUCHI

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Abstract. We extend the Hardy-Littlewood duality theorem to any locally compact abelian group G, namely, if $L^q(G)$ $(2<q<\infty)$ has the upper majorant property, then $L^p(G)$ has the lower majorant property, $p^{-1}+q^{-1}=1$. This settles the question of exactly which $L^p(G)$ has the lower majorant property.

1. Introduction. Let G be a compact abelian group. For $f, g \in L^1(G)$, we say as in [5] that g is a majorant of f if and only if $|\hat{f}| \leqq \hat{g}$. Let $1 \leqq p \leqq \infty$. We say as in [2] that $L^p(G)$ has the upper majorant property (UMP) if and only if there is a constant A_p such that

$$||f||_p \leqq A_p ||g||_p$$

whenever $f, g \in L^p(G)$ and g is a majorant of f. We say also as in [2] that $L^p(G)$ has the lower majorant property (LMP) if and only if there is a constant B_p such that every $f \in L^p(G)$ has a majorant $g \in L^p(G)$ for which

$$||g||_p \leqq B_p ||f||_p .$$

The majorant problem is to determine for which p the space $L^p(G)$ has the UMP or the LMP. To exclude trivialities we assume throughout that G is infinite. The problem was initiated by Hardy and Littlewood [5] and solved partially by them for the torus group T. The problem in the general compact abelian case has now been completely solved, collectively by Boas [2], Bachelis [1], and Fournier [4]. (See also Shapiro [10].) The results can be summarized in the following theorems.

THEOREM A. *$L^p(G)$ has* UMP *if and only if p is an even integer or ∞; and when $L^p(G)$ has the* UMP *the constant is* 1.

THEOREM B. *$L^p(G)$ has the* UMP *if and only if $L^q(G)$ has the* LMP, *with the same constant,* $(q^{-1} + p^{-1} = 1)$.

As an immediate consequence of these one also has

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