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ALGEBRAIC CLASSIFICATION OF DIFFEOMORPHISMS

R. E. STONG

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1. Introduction. Clearly, one would like to classify diffeomorphisms of manifolds, and several people have tried to do so up to cobordism. Recently Kreck [4] settled this in the oriented case. Since the cobordism problem seems so hard, it is then reasonable to simplify further by reducing to the corresponding algebraic cobordism problem. This algebraic cobordism problem can be completely solved.

Briefly, let $\mathfrak{N}_n(\mathrm{Iso})$ denote the cobordism group of n dimensional Poincaré algebras M together with an isomorphism $f: M \to M$, and let $\mathfrak{N}_n$ denote the unoriented cobordism group. Then:

a) For $n = 0$, $\mathfrak{N}_n(\mathrm{Iso})$ is the Z_2 vector space with base given by the mod 2 cohomology of sets with $2j + 1$ points permuted cyclically (for all $j \geqq 0$).

b) For n odd, $\mathfrak{N}_n(\mathrm{Iso})$ is isomorphic to $\mathfrak{N}_n$, and

c) For n even, $\mathfrak{N}_n(\mathrm{Iso}) \cong \mathfrak{N}_n \oplus A$ where A is a Z_2 vector space with a basis in 1-1 correspondence with the irreducible polynomials

$$p(x) = x^{2j} + a_1x^{2j-1} + \cdots + a_{2j-1}x + 1$$

over Z_2 which are symmetric, i.e., satisfy $a_{2j-i} = a_i$.

Relating this back to diffeomorphisms, it will be shown that every class in $\mathfrak{N}_*(\mathrm{Iso})$ comes from a diffeomorphism, using a proof by Charles Giffen. Finally it is noted that algebraically trivial diffeomorphisms need not be boundaries, as diffeomorphisms. For $n \geqq 3$ there is a non-trivial kernel.

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2. Dimension zero. The preliminaries and precise definitions for Poincaré algebras and Lefschetz algebras were first given by Brown and Peterson [1]. We will not insult the reader by repeating all of the formalism. The reader may also find it convenient to look at [6] Section 5 and at [7].

All Poincaré algebras considered here will be over the field Z_2. An