

# THE DUAL SPACE OF THE SPACE BMO FOR A STOCHASTIC POINT PROCESS

HIROAKI MORIMOTO

(Received October 23, 1978, revised March 13, 1979)

**1. Introduction.** Let  $(\Omega, F, P)$  be a complete probability space endowed with a non-decreasing right continuous family  $(F_t)_{t \geq 0}$  of sub  $\sigma$ -fields of  $F$  with  $F = \bigvee_{t \geq 0} F_t$  such that  $F_0$  contains all null sets. Let  $\lambda$  be a non-negative predictable process such that  $P\left(\int_0^t \lambda_s ds < \infty\right) = 1$  for all  $t$ . An  $F_t$ -adapted process  $N = (N_t)$  is called a stochastic point process with the intensity  $\lambda$  if  $N$  has right continuous paths taking values in  $Z_+ = \{0, 1, 2, \dots\}$  with  $N_0 = 0$ ,  $\Delta N_t = N_t - N_{t-} = 0$  or  $1$ , and if  $\hat{N}_t = N_t - \int_0^t \lambda_s ds$  is a local martingale. Throughout, we assume that the stochastic point process  $N$  with the intensity  $\lambda$  satisfies the following conditions:

(S)  $F_t = \sigma(N_s, s \leq t)$ , i.e.,  $F_t$  is the completion of the  $\sigma$ -field generated by  $(N_s, s \leq t)$ ,

(B)  $\hat{N}$  belongs to the space BMO.

Then we can define the finite measure  $\mu$  on the  $\sigma$ -field  $\mathcal{E}$  of all predictable subsets  $A$  of  $[0, \infty) \times \Omega$  by

$$(1) \quad \mu(A) = E\left[\int_0^\infty I_A \lambda_s ds\right].$$

We shall adopt the following notations and definitions:

$$(2) \quad L^1(\mu) \text{ denotes the set of all predictable processes } f \text{ with } \|f\|_{L^1(\mu)} < \infty;$$

$$(3) \quad L^\infty(\mu) = \{f \in L^1(\mu); \|f\|_{L^\infty(\mu)} < \infty\};$$

$$(4) \quad \Phi \text{ denotes the set of all real valued set functions } \nu \text{ on } ([0, \infty) \times \Omega, \mathcal{E}, \mu) \text{ such that } \nu(A \cup B) = \nu(A) + \nu(B) \text{ if } A, B \in \mathcal{E} \text{ and } A \cap B = \emptyset, \\ \|\nu\| = \sup \left\{ \sum_{i=1}^n |\nu(A_i)|; \{A_1, \dots, A_n\} \text{ is a measurable partition of } [0, \infty) \times \Omega \right\} < \infty, \text{ and } \nu(A) = 0 \text{ if } A \in \mathcal{E} \text{ and } \mu(A) = 0;$$

$$(5) \quad U(f)(t) = \int_0^t f_s d\hat{N}_s \quad \text{for } f \in L^1(\mu);$$