Tôhoku Math. Journ.
32 (1980), 235-254.

EXISTENCE OF PERIODIC SOLUTIONS AT RESONANCE FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

Dedicated to Professor Taro Yoshizawa on his sixtieth birthday

MAX ASHKENAZI AND SHUI-NEE CHOW*

(Received June 6, 1979, revised October 18, 1979)

0. Introduction. The purpose of this paper is to study bifurcation of periodic orbits from the equilibrium for differential equation with time delays. One of the most important results for such problems is the Hopf bifurcation theorem. (See, for example, Chaffee [2], Hale [6], Chow and Mallet-Paret [4].) We are interested in similar problems in the case that there are time periodic perturbations. Such problems have been discussed in Perello [9], Hale [8, Chapter 9], Ashkenazi [1]. Similar problems have been encountered in the study of epidemic models [11]. Numerical studies indicate that instability may occur even if time periodic perturbation is small. On the other hand, if there are only autonomous perturbations, then a stable periodic solution will occur. We will give a partial answer to these phenomena by showing how the small parameter in Hopf's bifurcation theorem interacts with the periodic perturbation. Our main result (Theorem 4.1) shows how one can determine the regions in which one of the parameters is more dominant. We do not give a stability analysis for the periodic orbits bifurcating from the equilibrium.

Our approach to the above is in the spirit of [3]. In fact, Hale [6] called this the restricted unfolding approach. Here, we begin with a specific parametrized family of bifurcation equations, (a two parameter family of equations in this paper). Even though it may be possible to use theorems such as Malgrange-Weierstrass Transs preparation theorem to reduce the equations to a normal form, this may not be the best way for the problem. In our case, we have a two-parameter family of equations on $\boldsymbol{R}^d$ (Euclidean d-dimensional space). The normal form may envolve a large number of parameters which may be difficult to be identified with the original parameters. Thus, we use techniques such as scaling and the implicit function theorem to obtain quite precise information about the problem. The disadvantage in this approach

* Partially supported by NSF Grant MCS 76-06739