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SPECTRAL LITTLEWOOD-PALEY DECOMPOSITIONS

GARTH I. GAUDRY

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1. Introduction. Let G be a compact abelian group with dual $\hat{G}$, and suppose that E is a subset of $\hat{G}$. Suppose that $(\varDelta_j)_0^\infty$ is a *decomposition* of E, i.e., that each $\varDelta_j$ is a subset of E, the $\varDelta_j$ are pairwise disjoint, and that $\bigcup \varDelta_j = E$. We say that $(\varDelta_j)$ is a *Littlewood-Paley* (*or* LP) *decomposition* of E if, for every p in $(1, \infty)$ there is a pair of positive constants A_p and B_p such that

$$A_p \| f \|_p \leqq \| (\sum | S_{\varDelta_j} f |^2)^{1/2} \|_p \leqq B_p \| f \|_p \tag{1}$$

for all trigonometric polynomials f with spectrum in E. Here $S_{\varDelta_j} f$, which we shall frequently denote $S_j f$, is the partial sum of the Fourier series of f over $\varDelta_j$. The function $(\sum | S_j(f) |^2)^{1/2}$ is denoted $S(f)$.

When $G = \boldsymbol{T}$, and $E = \boldsymbol{Z}$, classical theorems of Littlewood and Paley furnish examples of nontrivial LP decompositions: e.g., the collection of "dyadic intervals" constitutes such a decomposition; from this basic example, many others can be built up. See [2].

Now if $(\varDelta_j)$ is a decomposition of E, $p > 2$, and each $(\varDelta_j)$ is a *singleton* set, then the inequality (1) amounts to the statement that E is a $\Lambda(p)$ set. In the opposite vein, if $(\varDelta_j)$ is an LP decomposition of E and F is a set formed by selecting at most one element from each $\varDelta_j$, then F is a $\Lambda(p)$ set for every p. Given the extent of the literature on $\Lambda(p)$ sets, it seems natural to attempt to give examples of groups $\hat{G}$, proper subsets E of $\hat{G}$ and associated LP decompositions $(\varDelta_j)$ of E.

As just indicated, this can be done trivially when E is a $\Lambda(p)$ set for all p. Another way is to take an LP decomposition of a group $\hat{G}$ and then let E be the union of all but one of the sets of that decomposition. Our aims should therefore be stated more precisely: we wish to produce sets E, and associated decompositions $(\varDelta_j)$, such that (i) $(\varDelta_j)$ is an LP decomposition of E; and (ii) ξ_E, the characteristic function of E, is a (Fourier) multiplier of L^p for no p other than 2. Note that if (δ_j) is an LP decomposition of $\hat{G}$, then the characteristic function of each δ_j is a Fourier multiplier of L^p $(1 < p < \infty)$.

In her paper [1], Bonami showed how to construct various classes of sets which are $\Lambda(p)$ for all p, and gave precise asymptotic estimates