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THE p-ADIC HURWITZ L-FUNCTIONS

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0. Introduction. Recently various p-adic analogues of arithmetic functions are constructed. Kubota and Leopoldt [4] constructed p-adic L-functions by interpolating the values at nonpositive integers of Dirichlet L-functions. Morita [6] constructed a p-adic analogue of the Hurwitz-Lerch L-function $L(s, a, b, \chi) = \sum_{n=1}^{\infty} \chi(n) b^n (a+n)^{-s}$ from the same point of view. In this paper, we construct a p-adic analogue of the Hurwitz L-function $L(s, a, \chi) = L(s, a, 1, \chi)$ as a power series at $s = 0$.

In §1, we calculate the higher derivatives at $s = 0$ of the complex Hurwitz L-function (Theorem 1). In §2, we obtain a lemma for p-adic interpolation. We construct in §3.1 p-adic analogues $\alpha_l^*(a, \chi)$ of the coefficients of the expansion at $s = 0$ of the complex Hurwitz L-function. We then define a p-adic function $\zeta_p(s, a, \chi)$ by

$$\zeta_p(s, a, \chi) = \sum_{l=0}^{\infty} \alpha_l^*(a, \chi) s^l$$

and show in §3.2 that this function coincides with Morita's p-adic analogue in the case of $b = 1$.

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1. Complex Hurwitz L-functions. 1.1. We denote by $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{C}$ the fields of rational numbers, real numbers and complex numbers, respectively. We denote by $\operatorname{Re} s$ the real part of s.

Let χ be a Dirichlet character with conductor f, and let $L(s, \chi)$ be the Dirichlet L-function for the character χ. Put $\delta_\chi = 1$ if χ is trivial, and $\delta_\chi = 0$ otherwise. We define complex numbers $\beta_l(\chi)$ $(l \geqq 0)$ by

$$\beta_l(\chi) = (-1)^l (l!)^{-1} \sum_{a=1}^{m} \chi(a) \lim_{n\to\infty} \left[\sum_{k=0}^{n} \frac{\{\log(mk+a)\}^l}{mk+a} - \frac{\{\log(mn+a)\}^{l+1}}{m(l+1)} \right].$$

Then we have the following:

PROPOSITION 1. *For any positive multiple m of f, we have*