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ON SPHERICAL REPRESENTATION OF AN m-DIMENSIONAL SUBMANIFOLD IN THE EUCLIDEAN n-SPACE

Dedicated to Professor Shigeo Sasaki on his seventieth birthday

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1. Introduction. The spherical representation of a curve in the Euclidean 3-space is a representation on the unit sphere S^2 obtained with the use of tangent vectors. We consider a generalization of the notion of spherical representations to an m-dimensional submanifold in the Euclidean n-space. We denote a submanifold by (i, M) where M is an m-dimensional manifold and i is an immersion $i: M \to R^n$. If the spherical representation of (i, M) is regular, the image is an immersed submanifold of dimension $2m-1$ in the unit hypersphere of R^n. Any submanifold and its infinitesimal deformations we consider are assumed to be C^∞.

Let p be any point of M and $\{O\}_p$ be the origin of $T_p(M)$. To any half line of $T_p(M)$ from $\{O\}_p$ there corresponds a point of the unit hypersphere $S_0^{n-1}(1)$ of R^n. Taking all points p of M and all half lines of $T_p(M)$ from $\{O\}_p$ we get the spherical representation of (i, M).

For our purpose a little more precise description will be preferable. Any immersion i of M induces a Riemannian metric g on M and this determines the unit hypersphere $S_p(M)$ of $T_p(M)$. For any point (i, p) of (i, M) there exists just one m-dimensional tangent plane of (i, M) and in this tangent plane we can take a hypersphere of radius 1 and with center (i, p). Let us denote this hypersphere by $(i', S_p(M))$. Then for any point $q \in S_p(M)$ we have just one point (i', q) of R^n. Let O be the origin of R^n and OX be the oriented segment obtained by a parallel translation of oriented segment joining (i, p) to (i', q). Then X is a point of $S_0^{n-1}(1)$. Thus a mapping $s: S(M) \to S_0^{n-1}(1)$ is obtained such that $s(q) = X$ and we call s the spherical representation of (i, M), or the spherical representation of M induced by the immersion i.

In the present paper we consider only such cases that s is an immersion. Then s is called a regular spherical representation or a regular spherical map and its image a spherical image.

We take a compact orientable manifold M and consider the integral I of the volume element of the spherical image $s(S(M))$. I is a functional