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BOUNDEDNESS OF SOME OPERATORS COMPOSED OF FOURIER MULTIPLIERS

Dedicated to Professor Tamotsu Tsuchikura on his sixtieth birthday

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Introduction and notations. We will consider the transplantation theorems for the operators defined by Fourier multipliers.

We will use the notations and conventions as follows.

$\boldsymbol{R}^n$ denotes the n-dimensional Euclidean space and $\boldsymbol{Q}^n$ the unit cube $\{\theta = (\theta_1, \cdots, \theta_n) \in \boldsymbol{R}^n; -1/2 \leqq \theta_j < 1/2 \ (j = 1, \cdots, n)\}$. $\boldsymbol{Q}^n$ is identified with the n-dimensional torus $\boldsymbol{T}^n$. The dual of $\boldsymbol{R}^n$ is denoted by $\hat{\boldsymbol{R}}^n$ and the totality of all lattice points with integral coordinates in $\hat{\boldsymbol{R}}^n$ is denoted by $\boldsymbol{Z}^n$, which is the dual of $\boldsymbol{T}^n$.

The Fourier transform $\hat{f}$ of $f \in L^1(\boldsymbol{R}^n)$ is defined by

$$\hat{f}(\xi) = \int_{\boldsymbol{R}^n} f(x)e(-x\xi)dx ,$$

where $e(t) = \exp(2\pi it)$, $x = (x_1, \cdots, x_n) \in \boldsymbol{R}^n$, $\xi = (\xi_1, \cdots, \xi_n) \in \boldsymbol{R}^n$ and $x\xi = \xi x = \sum_{j=1}^n x_j\xi_j$. $g^\vee$ denotes the inverse Fourier transform of g. The Fourier coefficients $\hat{F}(m)$ $(m \in \boldsymbol{Z}^n)$ of $F \in L^1(\boldsymbol{T}^n)$ are defined by

$$\hat{F}(m) = \int_{\boldsymbol{Q}^n} F(\theta)e(-m\theta)d\theta .$$

For a bounded function λ on $\hat{\boldsymbol{R}}^n$, the operator T_λ is defined as follows. Let $f \in \mathscr{S}(\boldsymbol{R}^n)$, where $\mathscr{S}(\boldsymbol{R}^n)$ denotes the Schwartz class. $T_\lambda f$ is defined by

$$(T_\lambda f)(x) = \int_{\hat{\boldsymbol{R}}^n} \lambda(\xi)\hat{f}(\xi)e(x\xi)d\xi .$$

On the other hand, for an indefinitely differentiable periodic function $F \in C^\infty(\boldsymbol{T}^n)$, $\tilde{T}_\lambda F$ is defined by $(\tilde{T}_\lambda F)(\theta) = \sum_{m \in \boldsymbol{Z}^n} \lambda(m)\hat{F}(m)e(m\theta)$. The operators T_λ and $\tilde{T}_\lambda$ are usually called Fourier multiplier operators defined by λ and the sequence $\{\lambda(m)\}$, respectively. The extensions of T_λ and $\tilde{T}_\lambda$ to $L^p(\boldsymbol{R}^n)$ and $L^p(\boldsymbol{T}^n)$, respectively, will be denoted by the same notations.

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