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RIGIDITY OF SUPERMINIMAL IMMERSIONS OF COMPACT RIEMANN SURFACES INTO CP^2

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0. Introduction. The rigidity aspects of minimal hypersurfaces in a Euclidean space or a sphere have constantly drawn authors' attentions, about which we mention the recent conclusive result of Dajczer-Gromoll [12] which states that a complete minimally immersed hypersurface of dimension ≥ 4 in S^{n+1}, or in $\boldsymbol{R}^{n+1}$ if it dose not contain $\boldsymbol{R}^{n-3}$ as a factor, is rigid, even in $\boldsymbol{R}^N \supset \boldsymbol{R}^{n+1}$. On the other hand the failure of this theorem to hold in general for a Riemann surface is well-known, to which we should add the positive result of Barbosa [2] which says that a minimally immersed Riemann sphere in a sphere is rigid, that of Choi-Meeks-White [11] which asserts that a properly embedded minimal surface in $\boldsymbol{R}^3$ with more than one end is rigid, and that of Ramanathan [22] stating that for each compact Riemann surface minimally immersed in S^3, there are only finitely many other minimal immersions isometric to it.

Along another line of development, minimal immersions (especially the superminimal ones) of Riemann surfaces into CP^n have recently been extensively studied by several authors [6], [8], [13], [14], [15], [25]. It is the purpose of this paper to look into the rigidity problem for superminimal immersions of compact Riemann surfaces into CP^2; to the author's knowledge the only results of this kind are the rigidity theorem of Calabi [7] which says that a holomorphic curve (a special class of superminimal immersions) in CP^n is rigid, the rigidity of totally real superminimal immersions in CP^n in Bolton-Jensen-Rigoli-Woodward [3], and the rigidity of superminimal immersions of constant curvature in [3], Bando-Ohnita [4], and [10]. One different feature of minimal immersions of Riemann surfaces into CP^2 from those into S^3 is that the immersion is of (real) codimension 2, with respect to which the conclusion of rigidity would be harder to draw in general. However with the given holomorphic data which a superminimal immersion in CP^2 enjoys, we are able to assert the rigidity for large classes of superminimal immersions.

After some preliminaries in §1 on the structure of minimal immersions in CP^n through the work in Chern-Wolfson [8], [9], and Eschenberg-Gaudalupe-Tribuzy [15], we establish the result (Lemma 1) in §2 that infers that those points of a given superminimal immersion at which the curvature $K=4$ are exactly those ramified points of index ≥ 2 of either the holomorphic curve or the dual of the holomorphic curve (but

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