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POLYNOMIAL REPRESENTATIONS OF KNOTS*

ANANT R. SHASTRI†

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Abstract. In this paper, we show that every 1-dimensional knot-type in the three-dimensional Euclidean space has polynomial representations. We also write down specific polynomial expressions for the trefoil knot and the figure eight knot. This result strengthens a conjecture of Abhyankar, that there exist non-rectifiable polynomial embeddings of the complex line in the three-dimensional complex affine space.

1. Introduction. In 1977, at a Kyoto conference, Abhyankar [1] conjectured that there exist polynomial embeddings of the affine line A^1 in A^3, which are inequivalent under the polynomial automorphisms of A^3. If our field k is algebraically closed, then this is equivalent to saying that there exist ring-theoretic epimorphisms $\alpha, \beta: k[x, y, z] \to k[t]$ such that for no automorphism φ of $k[x, y, z]$, we have $\alpha \circ \varphi = \beta$. In support of this conjecture, Abhyankar further conjectured that the embeddings $\theta(m, n, l)$ given by

$$t \mapsto (t^m, t^n, t^l + t)$$

where the natural number m, n, l are such that none of them belongs to the additive semigroup generated by the other two are not equivalent to the standard embedding:

$$t \mapsto (t, 0, 0)\,,$$

i.e., $\theta(m, n, l)$ are *non-rectifiable*.

Subsequently, several authors have shown that, this latter conjecture is somewhat far-fetched! (For relevant literature, see [2], [3], [4], and [6].) To be precise, for instance, Creighero [3] showed that the embedding $\theta(m, n, l)$ is indeed equivalent to the standard embedding. Perhaps, after presenting a proof of the now famous EPIMORPHISM THEOREM, it was quite natural for Abhyankar to make the above conjecture, since the main step in the proof of this theorem is that if $x \mapsto f(t)$, $y \mapsto g(t)$ defines an epimorphism of $k[x, y]$ onto $k[t]$, then $\deg f$ divides $\deg g$ or $\deg g$ divides $\deg f$. (Here, $\mathrm{ch}(k) = 0$). However, a more important aspect of the results of these authors is that they obtain plausible candidates for non-tame automorphisms of A^3.

To bring in the topological point of view, let us from now on assume that the field

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