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BOUNDS OF AUTOMORPHISM GROUPS OF GENUS 2 FIBRATIONS

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Abstract. For a relatively minimal fibration of genus 2, the best bounds of the orders of its automorphism group, abelian automorphism group and cyclic automorphism group are obtained as a linear function of the self-intersection number of the canonical divisor.

It is well known that the automorphism group of a surface of general type is finite and bounded by a function of K^2 (cf. [1]). Since then, several authors worked on this subject and found better upper bounds of the group. Recently Xiao [11], [12] obtained a linear bound for this group. Hence it is natural to investigate the upper bounds for particular classes of surfaces. Here we are interested in the upper bounds of various automorphism groups of surfaces with genus 2 pencils. As a first step, in the present paper, we will study the upper bounds of automorphism groups of genus 2 fibrations.

We always assume that S is a smooth projective surface over the complex number field. A genus 2 fibration is a morphism $f: S \to C$ where C is a projective curve such that a general fiber of f is a smooth curve of genus 2.

DEFINITION 0.1. An automorphism of the fibration $f: S \to C$ is a pair of automorphisms $(\tilde{\sigma}, \sigma)$ with $\tilde{\sigma} \in \mathrm{Aut}(S)$, $\sigma \in \mathrm{Aut}(C)$ such that the diagram

$$\begin{array}{ccc} S & \xrightarrow{\tilde{\sigma}} & S \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle f} \\ C & \xrightarrow{\sigma} & C \end{array}$$

commutes.

The automorphism group of fibration f will be denoted by $\mathrm{Aut}(f)$. The main results of this paper are the following:

THEOREM 0.1. *Suppose S is a surface of general type over the complex number field with a relatively minimal genus 2 fibration $f: S \to C$. Then*

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