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PRINCIPAL SERIES WHITTAKER FUNCTIONS ON $Sp(2; \boldsymbol{R})$, II

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Abstract. We consider Whittaker model for generalized principal series representations of the real sympletic group of degree 2. We obtain an integral formula for the radial part of the vector of with an extreme K-type in the Whittaker model.

Introduction. In our previous papers [O], [M-O], we investigated Whittaker functions of the large discrete series representations, and of the principal series representations of the real symplectic group $Sp(2; \boldsymbol{R})$ of rank 2, respectively.

In this paper we shall obtain explicit integral formulae for the radial part of the Whittaker functions on $G = Sp(2; \boldsymbol{R})$, belonging to the principal series representations associated with the Jacobi parabolic subgroup P_1 of G.

Let (π, H_π) be an irreducible admissible representation. Denote by N a maximal unipotent subgroup of G. For a continuous character $\eta: N \to \boldsymbol{C}^*$ of N, let $C_\eta^\infty(N\backslash G)$ be the space of complex-valued C^∞-functions f on G satisfying

$$f(ng) = \eta(n) \cdot f(g) \qquad \text{for any} \quad n \in N\,, \ g \in G\,.$$

Consider $C_\eta^\infty(N\backslash G)$ as a $(\mathfrak{g}, K)$-module via the right regular action of G. Then the intertwining space

$$\mathrm{Hom}_{(\mathfrak{g},K)}(H_\pi, C_\eta^\infty(N\backslash G))$$

is the space of algebraic Whittaker vectors. When π is a principal series representation with a generic parameter μ of $\mathfrak{a}_{\boldsymbol{C}}^*$, the dimension of the above space is known and equals the order of the (little) Weyl group, i.e. 8 in our case (cf. Kostant [Kos, §5]). Here $\mathfrak{a}_{\boldsymbol{C}}^*$ is the dual of the complexification of the Lie algebra $\mathfrak{a}$ of A.

Choose a K-type (τ, V_τ), $\tau \in \hat{K}$, which occurs with multiplicity one in H_π, and let $i: V_\tau \hookrightarrow H_\pi$ be an injective K-homomorphism which is unique up to nonzero scalar multiple. Then we call the elements of the image of the restriction map

$$\mathrm{Hom}_{(\mathfrak{g},K)}(H_\pi, C_\eta^\infty(N\backslash G)) \to \mathrm{Hom}_K(V_\tau, C_\eta^\infty(N\backslash G)) \simeq C_\eta^\infty(N\backslash G) \otimes_K V_\tau^*\,,$$

Whittaker functions with K-type τ^* belonging to the representation π.

Now consider the standard maximal parabolic subgroup P_1 of G associated to the long simple root. In this paper we call this parabolic subgroup the Jacobi

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