

COMPUTABILITY-THEORETIC COMPLEXITY OF COUNTABLE STRUCTURES

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§1. Introduction. Computable model theory, also called effective or recursive model theory, studies algorithmic properties of mathematical structures, their relations, and isomorphisms. These properties can be described syntactically or semantically. One of the major tasks of computable model theory is to obtain, whenever possible, computability-theoretic versions of various classical model-theoretic notions and results. For example, in the 1950's, Fröhlich and Shepherdson realized that the concept of a computable function can make van der Waerden's intuitive notion of an *explicit* field precise. This led to the notion of a *computable* structure. In 1960, Rabin proved that every computable field has a computable algebraic closure. However, not every classical result "effectivizes". Unlike Vaught's theorem that no complete theory has exactly two nonisomorphic countable models, Millar's and Kudaibergenov's result establishes that there is a complete decidable theory that has *exactly two* nonisomorphic countable models with computable elementary diagrams. In the 1970's, Metakides and Nerode [58], [59] and Remmel [71], [72], [73] used more advanced methods of computability theory to investigate algorithmic properties of fields, vector spaces, and other mathematical structures. At the same time and independently, computable

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