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A NOTE ON QUOTIENT SPACES OF SUPERCOMPACT SPACES

By

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Abstract A space is called supercompact if it has an open subbase such that every cover consisting of elements of the subbase has a subcover consisting of two elements. In this paper we prove that the quotient space of a supercompact space obtained by identifying a finite set or a closed G_δ-set to a point is also supercompact thus answering a question of M. G. Bell.

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1. Introduction.

All spaces in this paper are assumed to be Hausdorff. *Supercompact spaces,* introduced by de Groot [4], are spaces X which possess an open subbase $\mathcal{G}$ such that every cover of X consisting of members of $\mathcal{G}$ has a subcover of at most 2 members. For our purposes it is more elegant to work with closed subbase. A collection of sets $\mathcal{G}$ is *linked* if every 2 members of $\mathcal{G}$ has a non-empty intersection. A collection of sets $\mathcal{G}$ is *binary* if every linked subcollection of $\mathcal{G}$ has a non-empty intersection. So, X is supercompact if and only if it has a binary closed subbase.

Many compact spaces, but not all, are supercompact. For example, all compact metric spaces are supercompact [3, 6]; all continuous images of compact ordered spaces are supercompact [2]. On the other hand, the author recently proved that every cluster point of a countable subset of a supercompact space is the limit of a nontrivial sequence [7]; therefore there exist many non-supercompact compact spaces. In 1990, Bell [1] gave a negative answer for the question of whether all dyadic spaces (=continuous images of 2^κ) are supercompact. In fact, Bell proved that there exists a supercompact subset $A\subset 2^{\omega_3}$ such that the quotient space obtained by identifying A to a point is not

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