A characterization of $PSU(3, 3^2)$ as a permutation group of rank 4

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1. Introduction

It is known that the simple unitary group $PSU(3, 3^2)$ of order 6048 has a representation as a primitive group of degree 36 with the stabilizer of a point isomorphic to the projective special linear group $PSL(3, 2)$ of order 168. This representation has rank 4 and subdegrees 1, 7, 7, $21=7\cdot 6/2$, and the orbitals of length 7 are paired with each other (for example, see Quirin [6, P. 224]).

The purpose of this note is to prove the following result, which is a supplement of section 2 of [5].

THEOREM. *Let (G, Ω) be a finite primitive permutation group of rank 4 such that the subdegrees are $1, k, k, k(k-1)/2$ and the orbitals of length k are paired with each other. Then $k=7$ and (G, Ω) is permutation-isomorphic to the simple unitary group $PSU(3, 3^2)$ acting by right multiplication on the cosets of its subgroup $PSL(3, 2)$.*

REMARK. By Proposition 3.6 of [5], if the stabilizer of a point acts doubly transitively on an orbit of length k, the assumption that the orbitals of length k are paired with each other is omitted.

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2. Notation and preliminaries

Our proof is quite elementary and only the familiarity with definitions and basic properties of Higman's intersection numbers ([4]) is assumed. Notation follows [4] and [5], but for convenience we rewrite below. The orbitals of length $1, k, k, l=k(k-1)/2$ are denoted by Γ_0, $\Gamma_1=\Delta$, $\Gamma_3=\Lambda$, $\Gamma_2=\Gamma$, respectively. Here we may take the orbitals so that $\Gamma_\alpha(a)^g=\Gamma_\alpha(a^g)$ for all $g\in G$ and $a\in\Omega$. The intersection numbers relative to an orbital Γ_α are defined by

$$\mu_{ij}^{(\alpha)}=|\Gamma_\alpha(b)\cap\Gamma_i(a)| \quad \text{for} \quad b\in\Gamma_j(a).$$

The following are fundamental relations among the $\mu_{ij}^{(\alpha)}$ and k, l.