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On infinitesimal projective transformations satisfying the certain conditions

By Kazunari YAMAUCHI

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§1. Introduction.

We consider the following problem

PROBLEM. *Let M be a compact Riemannian manifold with positive constant scalar curvature. If M adminits a nonisometric infinitesimal projective transformation, then is M a space of positive constant curvature?*

For this problem, the following results are known.

THEOREM A. *Let M be a complete Riemannian manifold with parallel Ricci tensor. If M admits nonaffine infinitesimal projective transformations, then M is a space of positive constant curvature.* [1].

THEOREM B. *Let M be a compact Riemannian manifold with consant scalar curvature K. If the scalar curvature is nonpositive, then an infinitesimal projective transformation is a motion.* [2].

THEOREM C. *Let M be a compact Riemannian manifold satisfying a condition* $\nabla_k K_{ji} - \nabla_j K_{ki} = 0$, $(K \neq 0)$, *where* ∇_k, K_{ji} *denote a covariant derivative and Ricci tensor, respectively. The projective Killing vector* v^h *can be decomposed uniquely as follows,*

$$v^h = w^h + q^h,$$

where w^h *and* q^h *are Killing vector and gradient projective Killing vector, respectively.* [2].

THEOREM D. *Let M be a compact Riemannian manifold satisfying a condition* $\nabla_k K_{ji} - \nabla_j K_{ki} = 0$, $(K \neq 0)$. *If M admits nonisometric infinitesimal projective transformations, then M is a space of positive constant curvature.* [2].

The purpose of this paper is to prove the following theorems

THEOREM 1. *Let M be a complete, connected and simply connectected Riemannian manifold with positive constant scalar curvature. If a projective Killing vector* v^h *is decomposable as follows,*