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On p-nilpotent groups with extremal p-blocks

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Throughout the present paper, G will represent a finite group, and p a fixed prime number. It is well known that

(I) if G is p-closed, then every p-block of G has full defect, and

(II) if G has the pTI-property, then every p-block of G has either full defect or defect zero.

Here, "G is p-closed" means that a Sylow p-subgroup of G is normal, and "G has the pTI-property" means that the intersection of two distinct Sylow p-subgroups of G is the identity. It is interesting to consider each converse of (I) and (II). In general, neither the converse of (I) nor of (II) is true. In fact, if H is a p-solvable group of p-length greater than 1, then $G=H/O_{p'}(H)$ has only one p-block, but G is neither p-closed nor has the pTI-property. In case $p=2$, several authors studied this problem ([1], [4], [5]). In this paper, we shall show that each converse of (I) and (II) is true if G is a p-nilpotent group. We shall use the following notations: $Z(G)$ is the center of G. Given $g\in G$, we put $x^{g}=gxg^{-1}$ for any $x\in G$, and $S^{g}=\{s^{g}|s\in S\}$ for any subset S of G.

For convenience' sake, we introduce the following definition.

DEFINITION. A group G is a *pFD-group* if every p-block of G has full defect. A group G is a *pFZD-group* if every p-block of G has either full defect or defect zero.

The following proposition is an immediate consequence of [7, Theorem 4], and plays an important role in our subsequent study.

PROPOSITION 1. *Let G be a p-nilpotent group with a normal p-complement N. Then G is a pFD-group if and only if, for every $x\in N$, $C_{G}(x)$ contains a Sylow p-subgroup of G.*

By making use of Proposition 1, we can easily obtain the following, which contains [1, Theorem 1].

THEOREM 1. *Let G be a p-nilpotent group. Then G is p-closed if and only if it is a pFD-group.*

PROOF. It suffices to prove the if part. We put $N=O_{p'}(G)$. If $x\in N$,