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Decomposition of convolution semigroups on Polish groups and zero-one laws

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Zero-one laws for infinitely divisible probability measures on a topological group G have quite a long history. (For a recent survey see the article [9] of A. Janssen.) Given a continuous convolution semigroup $(\mu_t)_{t\geq 0}$ of probability measures on G and a measurable subgroup H of G, one looks for conditions on $(\mu_t)_{t\geq 0}$ which yield $\mu_t(H)=0$ for all $t>0$ or $\mu_t(H)=1$ for all $t>0$. There are two classes of groups to which special attention has been given in this context: Locally compact groups; and topological vector spaces, in particular Banach spaces. But for technical reasons, on non-commutative groups mainly normal subgroups and normal convolution semigroups have been considered (for example see [8, 9, 12]).

In 1983 a new idea was introduced in this field by T. Byczkowski and A. Hulanicki [4]. In order to obtain a zero-one law for Gaussian semigroups $(\mu_t)_{t\geq 0}$ on a Polish group G, they defined the resolvent measure $\mu=\int_0^\infty e^{-t}\,\mu_t\,dt$ and dealt with the space $L^1(\mu)$ (instead of a space of continuous functions on G). But this is quite natural since the indicator function of H is μ-integrable but not continuous (unless H is open). A further step along these lines was taken by T. Byczkowski and T. Żak [5]. If $\mu_t(H)>0$ for all $t>0$, then there exist a continuous convolution semigroup $(\lambda_t)_{t\geq 0}$ on G supported by H and a bounded measure ρ on G supported by $\complement H$ such that the infinitesimal generator of $(\mu_t)_{t\geq 0}$ is the sum of the infinitesimal generators of $(\lambda_t)_{t\geq 0}$ and of the Poisson semigroup $(e(t\rho))_{t\geq 0}$ with exponent ρ (decomposition theorem). An unsatisfactory aspect of [5] is that (for technical reasons) only Polish groups of the type $G=F^\infty$ are admitted (where F is a second countable locally compact group).

Although only normal subgroups H have been considered in [4] and [5], it is possible to get rid of this restriction by application of the following results: 1. An estimation of the growth of $\mu_t(H)$ as t tends to 0 ([10], cf. Lemma 1.6. below). 2. Every continuous convolution semigroup $(\mu_t)_{t\geq 0}$ admits a Lévy measure ([13], cf. 1.3. below). Indeed, the rôle played by the Lévy measure in the context of zero-one laws, is well known (cf. [8, 9]).