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On the Existence of bounded Analytic Functions in a lacunary End of a Riemann surface.

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If a domain G in a Riemann surface R has a compact relative boundary ∂G, we call G an end. Let G be an end of a Riemann surface $\in O_g$. Suppose G has a boundary component $\mathfrak{p}$. The maximal number of linearly independent H. P. s (positive harmonic functions) vanishing on ∂G is called the H-dim of $\mathfrak{p}$. Let F be a closed set in G such that $G-F$ is connected. Let $G'(z, z_0)$: $z_0 \in G-F$ be a Green function of $G-F$. If $\overline{\lim_{z\to\mathfrak{p}}} G'(z, z_0)>0$, we say F is irregular at $\mathfrak{p}$. If there exists a sequence $\{\Gamma_n\}$ such that Γ_n consitst of a finite number of analytic curves separating $\mathfrak{p}$ from ∂G and

$$\overline{\lim_{n}} \operatorname*{Min}_{z\in\Gamma_n} G'(z, z_0)>0,$$

we say F is completely irregular at $\mathfrak{p}$. Further if every Γ_n consists of an analytic curve, we say F is completely thin at $\mathfrak{p}$. Evidently if G is a punctured disk : $\{0<|z|<1\}$, F is completely thin at $z=0$ if and only if F is irregular at $z=0$.

We proved

THEOREM[1] 1. *Let G be an end of a Riemann surface $\in O_g$ with a boundary component $\mathfrak{p}$ of H-dim$=\infty$. If F is completely thin at $\mathfrak{p}$,*

$$G-F\in O_{AB}.$$

For Riemann surfaces $\notin O_g$ analogous theorems[2] are discussed before. For examples.

There exists a Riemann surface $R\notin O_g$ with the following properties :

1) *R has no singular boundary points with respect to Martin's topology.*

2) *There exists a boundary point p which is a singular point of second kind with respect to N-Martin's topology such that*

$$G \overset{N}{\ni} p \text{ implies } G\in O_{AB},$$

where $G \overset{N}{\ni} p$ means G is a fine neighbourhood of p with respect to the