Hokkaido Mathematical Journal Vol. 16 (1987), p. 29～42

Characterization of Poisson Integrals of Vector-Valued Functions and Measures on the Unit Circle

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(Received April 8, 1985, Revised June 30, 1986)

Introduction.

An answer to the question whether, for a given complex-valued harmonic function f in the open unit disk D, there exists a finite measure on $[-\pi, \pi]$ (i. e. on the unit circle Π) such that f is the Poisson integral of this measure can be given in terms of the family of functions $\{f_r;\ 0\leq r<1\}$ defined on the unit circle by

(1) $$f_r : e^{i\theta}\mapsto f(r\,e^{i\theta}),\ \theta\in[-\pi, \pi].$$

Namely, such a measure exists if and only if there exists a constant α, independent of r, such that

$$\int_{-\pi}^{\pi}|f_r(e^{i\theta})|\,d\theta\leq\alpha,$$

for each $0\leq r<1$. This condition means that the linear maps Φ_r, $0\leq r<1$, from the space $C(\Pi)$ of continuous functions on the unit circle (equipped with the uniform norm) into the complex numbers defined by

(2) $$\Phi_r(\psi)=\int_{-\pi}^{\pi}\psi(\theta)f_r(e^{i\theta})\,d\theta,\ \psi\in C(\Pi),$$

map the unit ball of this space into a bounded set independent of r.

Just as well known is the criterion that f is the Poisson integral of an integrable function on Π if and only if the net of functions $\{f_r;\ 0\leq r<1\}$ is Cauchy in the sace $L^1(\Pi)$.

If f is a harmonic function in D, but now with values in a Banach space X, in which case the family of functions $\{f_r;\ 0\leq r<1\}$ also assumes its values in the space X, then it is natural to ask whether the classical results for numerical-valued functions have vector analogues which characterize f as the Poisson integral of an X-valued measure or integrable function on the unit circle. The aim of this note is to show that this is indeed the case.

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* Supported by an Australian-American Fulbright Award while visiting the University of Illinois at Urbana-Champaign. The author wishes to thank the referee for some valuable remarks and suggestions.