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Complex powers of a class of pseudodifferential operators in R^n and the asymptotic behavior of eigenvalues

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§ 0. Introduction

In the previous paper [2], we constructed complex powers for some hypoelliptic pseudodifferential operators P in $OPL^{m,M}(\Omega;\Sigma)$ (for the notation, see Sjöstrand [18]) on a compact manifold Ω of dimension n without boundary and examined the asymptotic behavior of the eigenvalues of P. Here the principal symbol vanished exactly to M-th order on the characteristic set Σ of codimension d in $T^{*}\Omega\backslash 0$. The hypoellipticity of these operators is well known by Boutet de Monvel [3] for $M=2$ and Helffer [6] for general M. Moreover Menikoff-Sjöstrand [11], [12], [13], Sjöstrand [19] and Iwasaki [9] studied the asymptotic behavior of eigenvalues of P under various assumptions on Σ in the case $M=2$. Their methods are based on the constructions of heat kernel and an application of Karamata's Tauberian theorem. For general M, Mohamed [14], [15] and [16] gave the asymptotic formula for the eigenvalues of P by using Carleman's method in which the Hardy-Littlewood Tauberian theorem was used.

However the method in [2] was essentially due to Minakshinsundaram's method (c. f. Seeley [17] and Smagin [20]). The essentials of the theory in [2] were as follows: At first we construct complex powers $\{P^{z}\}_{z\in C}$ of P. When the real part of z is negative and $|z|$ is sufficiently large, P^{z} is of trace class and the trace is extended to a meromorphic function in C which is written by Trace(P^{z}). Secondly we examine the first singularity of Trace(P^{z}). Finally we apply the extended Ikehara Tauberian theorem. (See [2: Lemma 5.2] and Wiener [21]). Here since Trace(P^{z}) is a meromorphic function in C, we call the pole with the smallest real part the first singularity throughout this paper. More precisely, denoting the counting function of eigenvalues by $N(\lambda)$, the first term of the asymptotic behavior of $N(\lambda)$ as λ tends to infinity is closely related to the position and the order of the pole at the first singularity. In the case where $n/m=d/M$, the first singularity situates at $z=-n/m$ and is a double pole and then we have for a constant c