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Well posedness for quasi-linear hyperbolic mixed problems with characteristic boundary

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§ 1. Introduction and results.

Let G be a domain in $R^{n}(n\geq 2)$ with smooth and compact boundary ∂G. We consider the mixed problem for symmetrizable hyperbolic system P:

$$\text{(P, B)}\quad \begin{cases} Pu\equiv(D_{t}+\sum_{j=1}^{n}A_{j}D_{j}+C)u=f & \text{in } [t_{1}, t_{2}]\times G, \\ Bu=g & \text{on } [t_{1}, t_{2}]\times\partial G, \\ u(t_{1}, x)=h & \text{for } x\in G, \end{cases}$$

and seek a solution $u\in X_{p}([t_{1}, t_{2}]\,;G)$ with a nonnegative integer p under the conditions (I)～(V) described below, where $D_{t}=-i\frac{\partial}{\partial t}\equiv D_{0}$, $D_{j}=-i\frac{\partial}{\partial x_{j}}$, $X_{p}([t_{1}, t_{2}]\,;G)=\bigcap_{i=0}^{p}C^{i}([t_{1}, t_{2}]\,;H^{p-i}(G))\equiv X_{p}(G)\equiv X_{p}$ and $C^{i}([t_{1}, t_{2}]\,;H^{k}(G))$ is of class C^{i} on $[t_{1}, t_{2}]$ to $H^{k}(G)$, k-th order Sobolev space in G.

(I)(i) The A_{j} and C are $m\times m$ matrices belonging to $X_{q}([t_{1}, t_{2}]\,;G)$ where $q=\max\left(p, \left[\frac{n}{2}\right]+2\right)$.

(ii) There exist $a_{0}\geq 1$ and A_{0} with $D_{t}A_{0}\in X_{q-1}([t_{1}, t_{2}]\,;G)$ for $i\geq 0$ such that A_{0} and the $A_{0}A_{j}$ are hermitian and $a_{0}^{-1}\leq A_{0}\leq a_{0}$.

(II) $B(t, x)$ is a $d^{+}\times m$ C^{∞}-matrix of contant rank d^{+}.

(III) The boundary matrix A_{ν} is of constant rank d less than m on ∂G, so that ∂G is characteristic for P. Here for x near ∂G $A_{\nu}=\sum_{j=1}^{n}A_{j}\nu_{j}$ and $\nu(x)=(\nu_{1}, \cdots, \nu_{n})$ stands for the unit inward normal to ∂G at the boundary point nearest to $x\in\bar{G}$.

(IV) The kernel B is maximally nonpositive for $A_{0}A_{\nu}$ on ∂G, i.e.,

$$(1.1)\qquad A_{0}A_{\nu}u\cdot u\leq 0 \qquad \text{for } u\in\ker B \text{ on } \partial G$$

and ker B is a maximal subspace obeying this property. Note that this implies the number of positive eigenvalues of A_{ν} is d^{+} on ∂G.

Now, since A_{ν} is of rank d on ∂G and $A_{0}A_{\nu}$ are hermitian, there exist

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