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A Santalo's formula in L-P

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Abstract It is show that a formula by Santalö on hyperbolic space of curvature -1 holds for Lorentz-Poincarë upper half space with curvature 1.

Introduction We call *L-P* plane, or simply *L-P*, relating to Lorentz-Poincarë, to the upper half space with the metric $ds^2=\dfrac{dx^2-dy^2}{y^2}$. The curvature of the *L-P* plane is 1.

If z is a complex variable, the group $SL(2)$ acts on the upper half plane $\mathrm{Im}(z)>0$ as the transformation group.

$$z'=\frac{az+b}{pz+q} \qquad aq-bp=1$$

Where a, b, p, q are real numbers. This is the classical Poincarë model for non-euclidean hyperbolic geometry. In the first section we introduce the double numbers, see [1], [7] and [8]. The referee observed that the reference [6], pag. 166, is appropiate. We show that substitution in the above transformation of the complex variable by a double number variable we obtain the Lorentz-Poincarë geometry. We also find relationship between double numbers, curvature and geodesics. Our main results is the integral formula in the third section.

Along the second section we obtain different expressions for the density of points, pair of points, geodesics, pair of geodesics, and kinematic density as is customary in integral geometry. Some of them will be used in the following section.

1. Double numbers in L-P

Let *L-P* plane be the upper half plane of Lorentz-Poincarë that means, the upper half plane $y>0$ with the metric

$$ds^2=\frac{dx^2-dy^2}{y^2} \tag{1}$$

Considering [1] and [7], we find an interesting relation between this metric and the so-called double numbers.

As a generalization of complex numbers, Benz, [1], and Yaglom, [7],