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Note on even tournaments whose automorphism groups contain regular subgroups

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§ 1. Introduction

A (0, 1)-matrix A of degree v is called a tournament of order v if A satisfies the following equation

(1) $\qquad A+A^{t}+I=J,$

where t denotes the transposition, and I and J are the identity and all one matrices of degree v respectively. In other words, a tournament is the adjacency matrix of a complete asymmetric digraph.

A tournament A is called even if the inner product of any two distinct row vectors of A is even.

A permutation matrix P such that $P^{t}AP=A$ is called an automorphism of A. The multiplicative group $\mathfrak{G}(A)$ of all automorphisms of A is called the automorphism group of A.

In the present note we consider a tournament A such that $\mathfrak{G}(A)$ contains a regular subgroup $\mathfrak{G}$. In previous two notes we considered the case where $\mathfrak{G}$ is cyclic (**1**) and (**2**). In such a case A is called a cyclic tournament. We obtained the following result in (**2**).

THEOREM. *An even cyclic tournament of order* v *exists if and only if* v *satisfies one of the following conditions*: (i) v *is congruent to* 3 *modulo* 8 *and the order of* 2 *modulo every prime divisor of* v *is singly even, where an even integer* n *is called singly even if* n *is not divisible by* 4; (ii) v *is cogruent to* 1 *modulo* 8 *and the order of* 2 *modulo every prime divisor of* v *is odd.*

Now since $\mathfrak{G}$ is regular, we label rows and columns of A by elements of $\mathfrak{G}$ so that

(2) $\qquad A=(A(a, b))$, where a and b are elements of $\mathfrak{G}$,

and

(3) $\qquad A(ac, bc)=A(a, b)$, where c runs over all elements of $\mathfrak{G}$.