Hokkaido Mathematical Journal Vol. 22(1993) p. 35-61

Types of the canonical isometric imbeddings of symmetric R-spaces

Dedicated to Professor Noboru Tanaka on his 60th birthday

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(Received February 20, 1992, Revised May 8, 1992)

Introduction.

In the history of Riemannian geometry, the rigidity problem of isometric imbeddings has been one of the major problems and has been studied by many authors.

Let f be an isometric imbedding of a Riemannian manifold M into a euclidean space R^m. f is called *rigid* if any other isometric imbedding of M into R^m can be written by a composite of f and a euclidean transformation of R^m. The rigidity problem is to determine whether a given isometric imbedding f is rigid or not. In his paper [**Tn**], N. Tanaka threw a new light upon the rigidity problem and made a great contribution to the progress of this problem.

Let f be an isometric imbedding of M into R^m. We define a differential operator Φ_{*f} by setting

$$\Phi_{*f}(u)=<df, du>,$$

where u is a differentiable map of M to R^m. A solution u of the equation $\Phi_{*f}(u)=0$ is called an *infinitesimal isometric deformation* of f. N. Tanaka proved that under the assumption that f is non-degenerate, there is a differential operator L associated with f, which is, in a sense, equivalent to the operator Φ_{*f} and the solution space of the equation $L\varphi=0$ is isomorphic with the space of infinitesimal isometric deformations of f.

It is noted that the operator L has a preferable property as a differential operator; the symbol of L is not necessarily degenerate, although the symbol of Φ_{*f} is necessarily degenerate. Therefore, through the operator L, the rigidity problem can be observed from a viewpoint of the differential equation.

In [**Tn**], N. Tanaka studied the case where the operator L is of elliptic type and f is infinitesimally rigid; an isometric imbedding f is called *infinitesimally rigid* if each solution of $L\varphi=0$ corresponds to an infinitesimal euclidean transformation of R^m. Applying the theory of elliptic differential equation, he established a global rigidity theorem for such